

Discussion 02

HOF, Environment Diagrams (for days)

6/28

1.2 (last part)

Walking through how to execute function calls

What will the following output?

```
def negate(f, x):  
    return -f(x)  
def square(n):  
    return n * n  
def double(n):  
    return 2 * n  
>>> negate(double, negate(square, -4))
```

Evaluate Operators and Operands

```
>>> negate(double, negate(square, -4))
```

Evaluate Operators and Operands

Evaluate the
operator

```
>>> negate(double, negate(square, -4))
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Evaluate Operators and Operands

We have defined the function with the name **negate** in global

Evaluate the operator

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Evaluate the operator

Evaluate the operands

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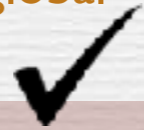
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Evaluate the operands

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Evaluate the operator

Evaluate Operators and Operands

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negate(double, negate(square, -4))

Evaluate the operator

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Evaluate Operators and Operands

We have defined the function with the name **negate** in global

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Evaluate the operands

square and -4 are both primitive

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negate(double, negate(square, -4))

Evaluate the operator

Evaluate the operands

Evaluate Operators and Operands

We have defined the function with the name **negate** in global

Evaluate the operator

We have defined the function with the name **double** in global

Evaluate the operands

square and -4 are both primitive

>>> negate(double, negate(square, -4))

We evaluated the operator, evaluated the operands. We are now ready for our first function call to negate. Note that this function call comes before the call to the negate on the outside of all the parenthesis (gray)

Evaluate the operator

Evaluate the operands



Evaluate Operators and Operands

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Evaluate the operator

Evaluate the operands



Execute the function call:

```
f1: negate [P = G]
  f: square
  x: -4
  RV: -16
```

```
f2: square [P = G]
  x: -4
  RV: 16
```

Evaluate Operators and Operands

We have defined the function with the name **negate** in global

Evaluate the operator

We have defined the function with the name **double** in global

Evaluate the operands

square and -4 are both primitive

>>>

negate(double,

-16

)

The function call returned 16, so we can replace the complicated looking operand with the value 16

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Evaluate the operator

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Execute the function call:

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Evaluate Operators and Operands

We have defined the function with the name **negate** in global

Evaluate the operator

We have defined the function with the name **double** in global

Evaluate the operands

square and -4 are both primitive

>>>

negate(double,

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The function call returned 16, so we can replace the complicated looking operand with the value 16

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Evaluate the operator

Evaluate the operands



Execute the function call:

```
f1: negate [P = G]
  f: square
  x: -4
  RV: -16
```

```
f2: square [P = G]
  x: -4
  RV: 16
```

Note: this is not a fully complete environment diagram (there are missing components)

The rules for executing a function call are:

1. Evaluate the operator
2. Evaluate the operands
3. Execute the body of the function

Note that these rules can be interrupted. In this example we were preparing to execute the first negate, but were interrupted in the process of evaluating it's operands. Sometimes it is necessary to complete another function call before completing the one we initially started

Execute the Function Call

Evaluate the operator

Evaluate the operands

>>> negate(double, -16)



Now that we know that values of the operands, we can execute the outer most function call

Execute the Function Call



Now that we know that values of the operands, we can execute the outer most function call

Execute the function call:

Execute the Function Call



Now that we know that values of the operands, we can execute the outer most function call

Execute the function call:

```
f1: negate [P = G]
  f: double
  x: -16
  RV: 32
```

```
f2: double [P = G]
  x: -16
  RV: -32
```

Execute the Function Call



Now that we know that values of the operands, we can execute the outer most function call

Execute the function call:

```
f1: negate [P = G]
  f: double
  x: -16
  RV: 32
```

```
f2: double [P = G]
  x: -16
  RV: -32
```

Solution: 32

What's different with HOF?

What's different between the code on the left and the code on the right? What will be printed when the code on the left is executed? What about the code on the right?

```
t = "surprise!"
def outer(t):
    def inner():
        print(t)
    return inner
outer("boo!")()
```

```
t = "surprise!"
def inner():
    print(t)
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t = "surprise!"
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Draw environment diagrams to see what's different

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t = "surprise!"
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Draw environment diagrams to see what's different

Global Frame

t: "surprise!"

outer: func outer(t) [P = G]

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t = "surprise!"
def outer(t):
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outer("boo!")()
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Draw environment diagrams to see what's different

Global Frame

t: "surprise!"
outer: func outer(t) [P = G]

f1: outer [P = G]

t: "boo!"
inner: func inner() [P = f1]
rv: inner

```
t = "surprise!"
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f1: outer [P = G]

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rv: inner

f2: inner [P = f1]


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Global Frame
t: "surprise!"
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f2: inner [P = f1]

f2: inner [P = G]

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f2: inner [P = f1]

f2: inner [P = G]

All inner does is print(t).
Since t is not defined in
the **local** frame, where
does inner find t?

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Python prints:

boo!

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Python prints:

surprise!

Environment Diagrams

Know the rules!

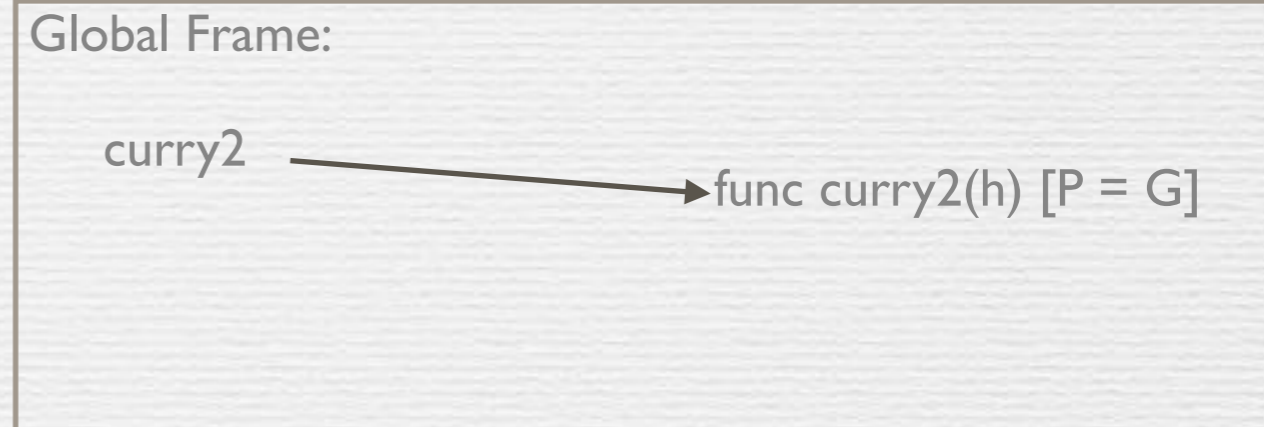
1. **Def** statements:
 1. create a new function whose parent is the current frame
 2. skip the body of the function
 3. bind the function to its name in the current frame
2. **Assignment** statements:
 1. evaluate the RHS
 2. bind the value of the RHS to the name on the LHS
 3. NOTE: names can only have one value per frame
3. **Function** calls:
 1. evaluate the operator
 2. evaluate the operands
 3. execute the body of the function

1.5 #1

```
def curry2(h):  
    def f(x):  
        def g(y):  
            return h(x, y)  
        return g  
    return f  
  
make_adder = curry2(lambda x, y: x + y)  
  
add_three = make_adder(3)  
  
five = add_three(2)
```

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Global Frame:

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1.5 #1

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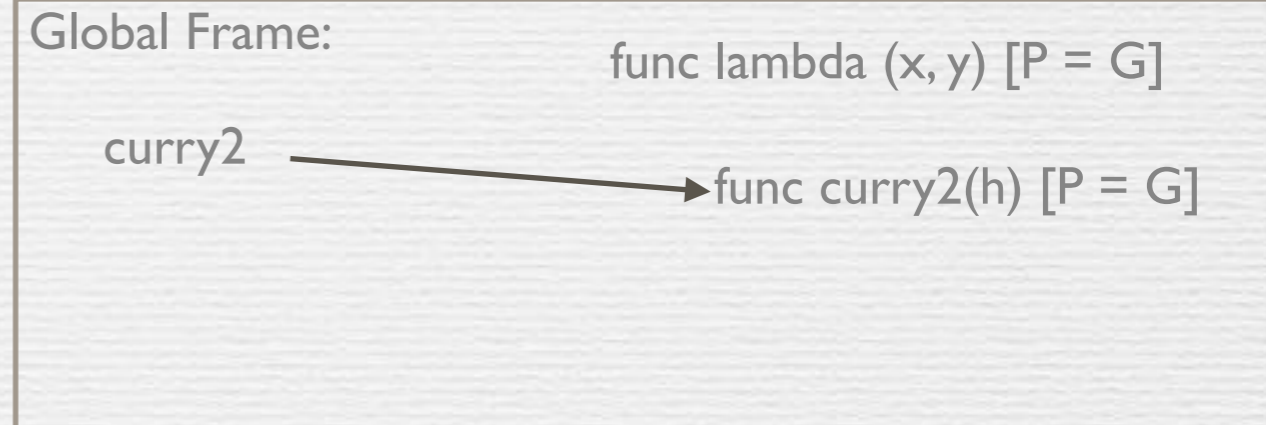
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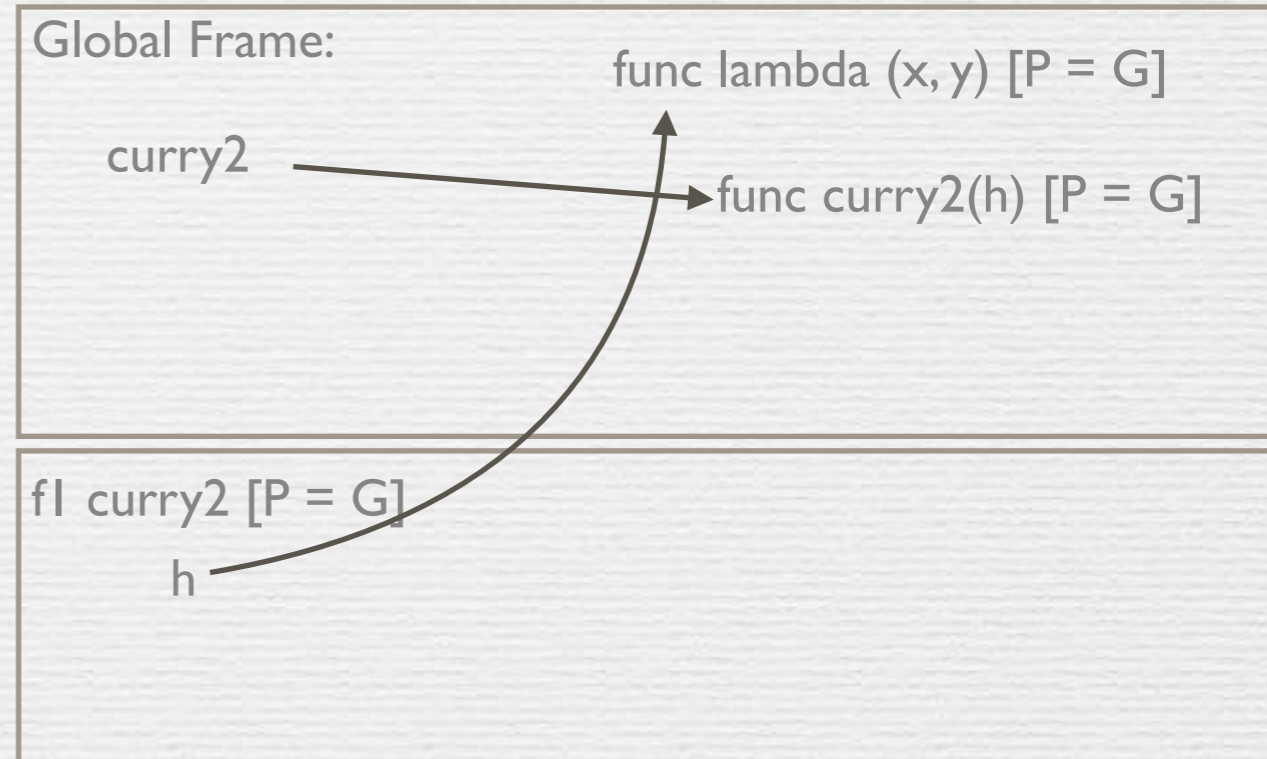


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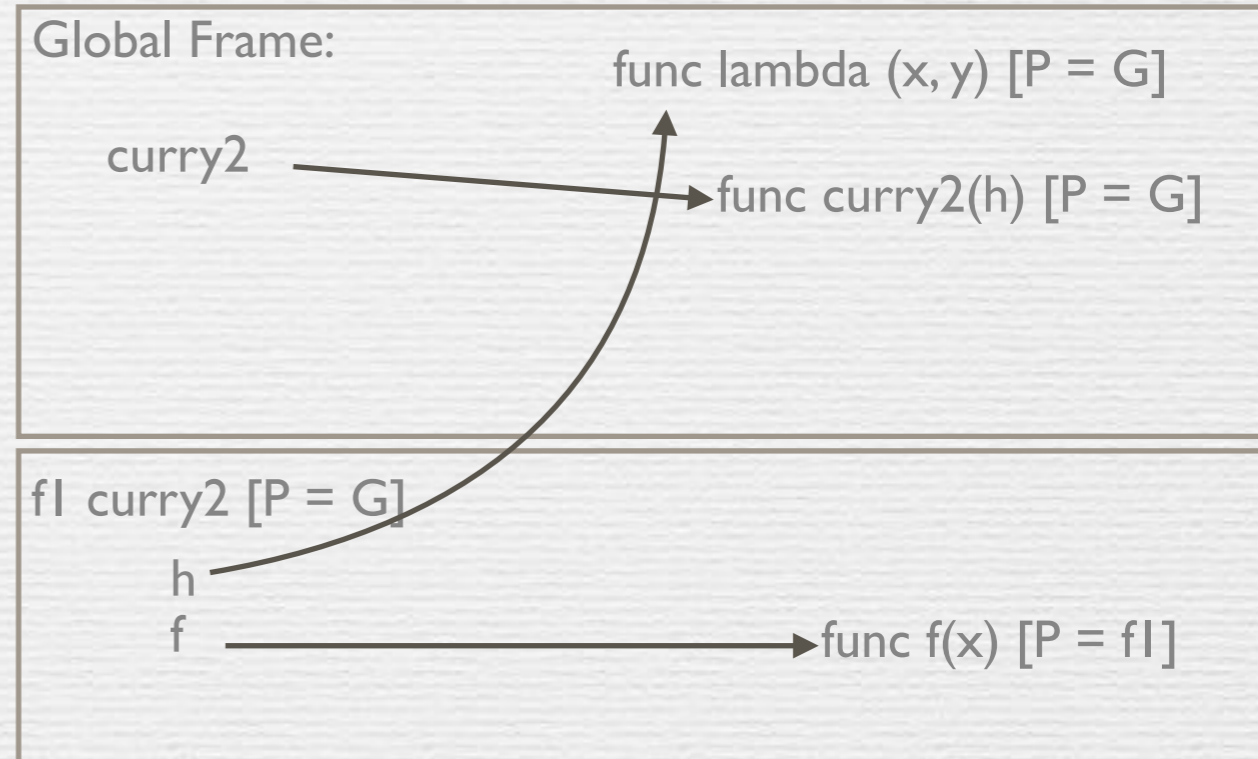


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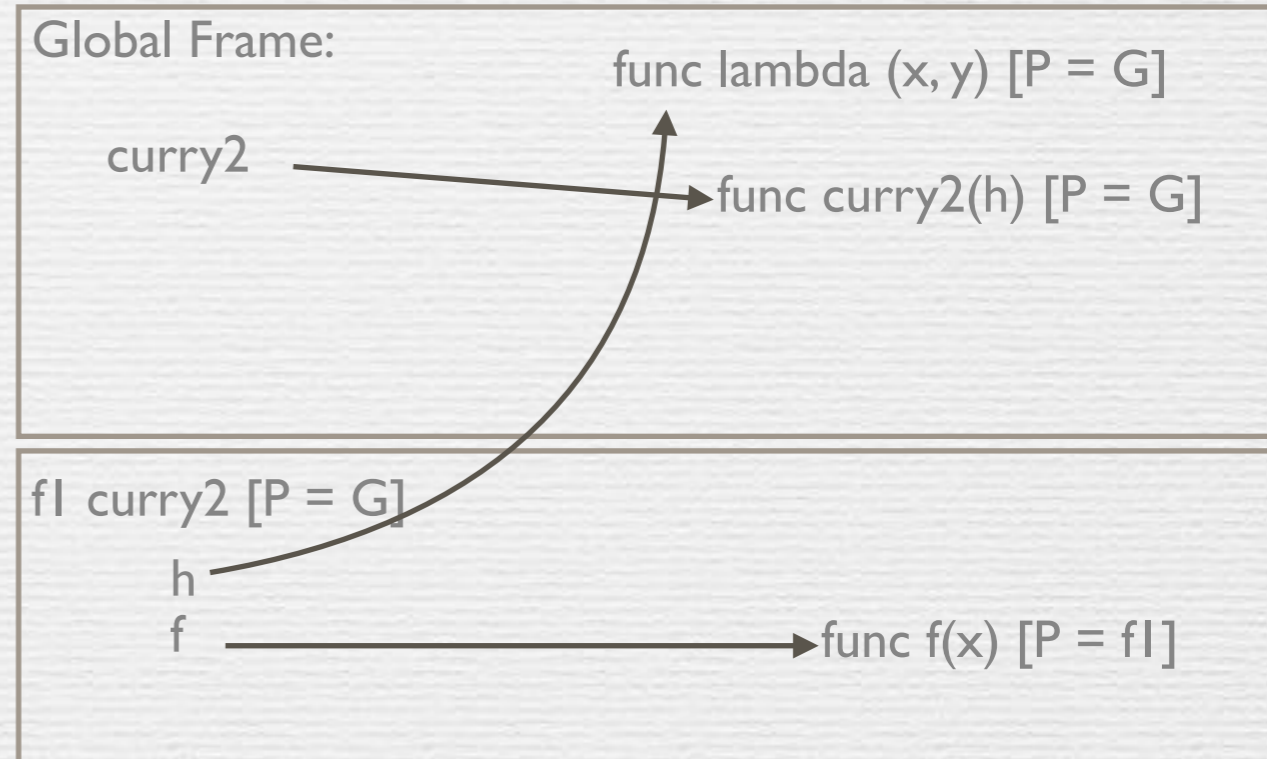


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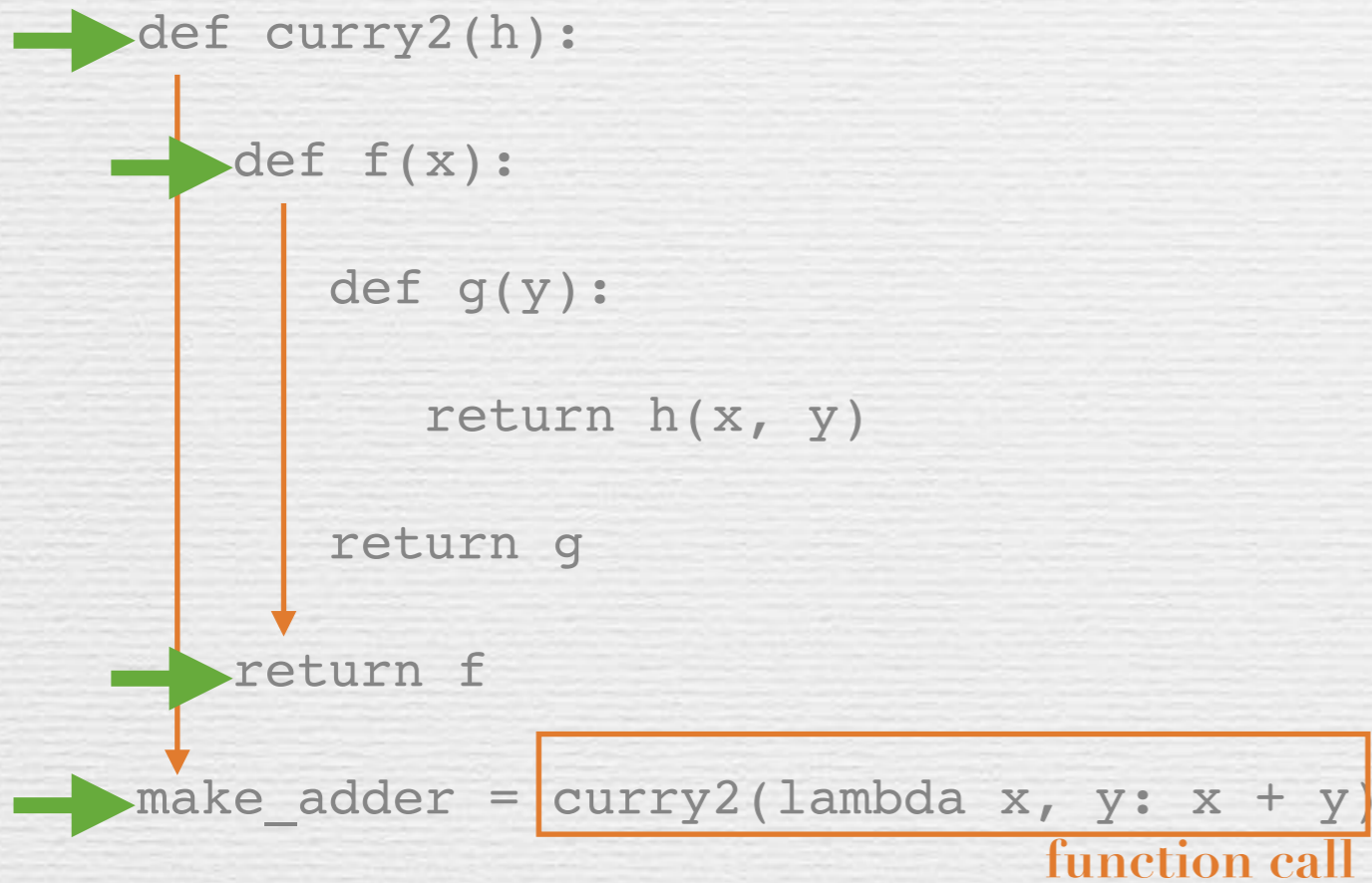
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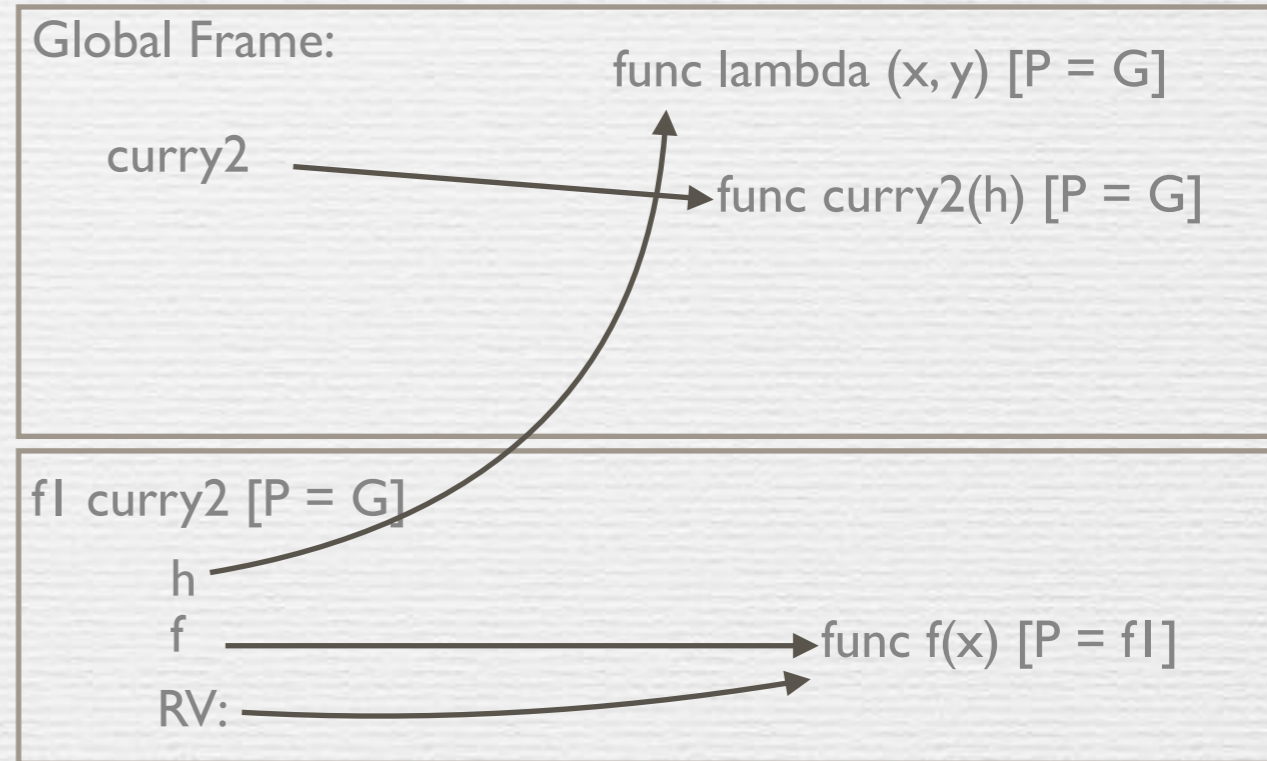


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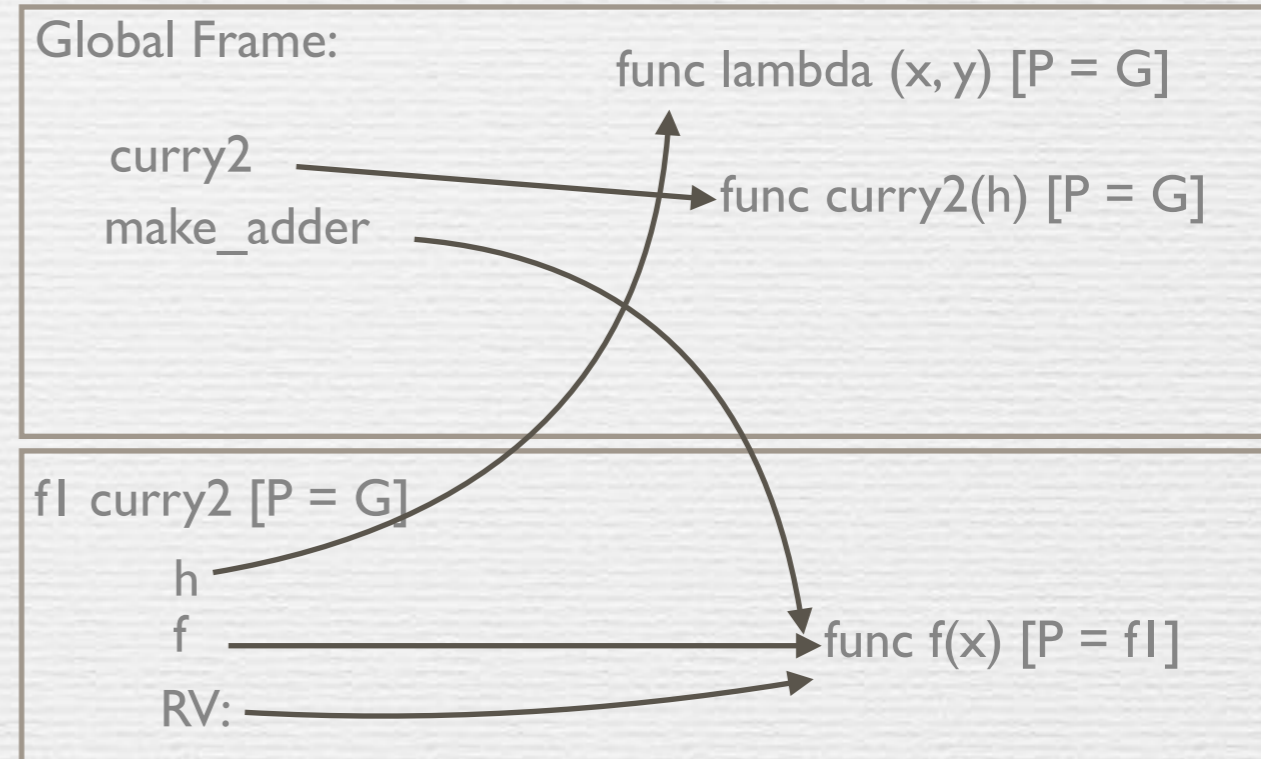
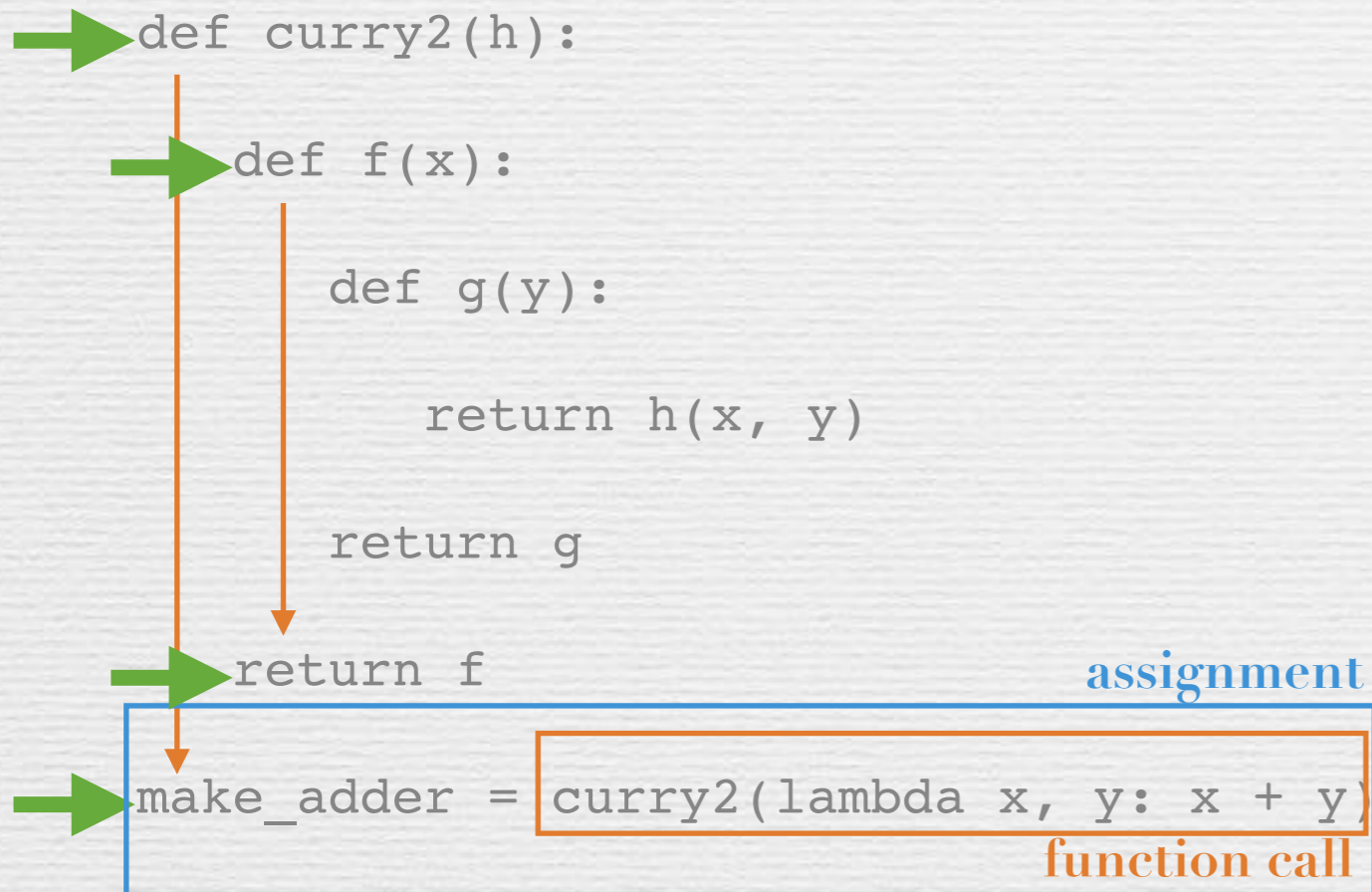


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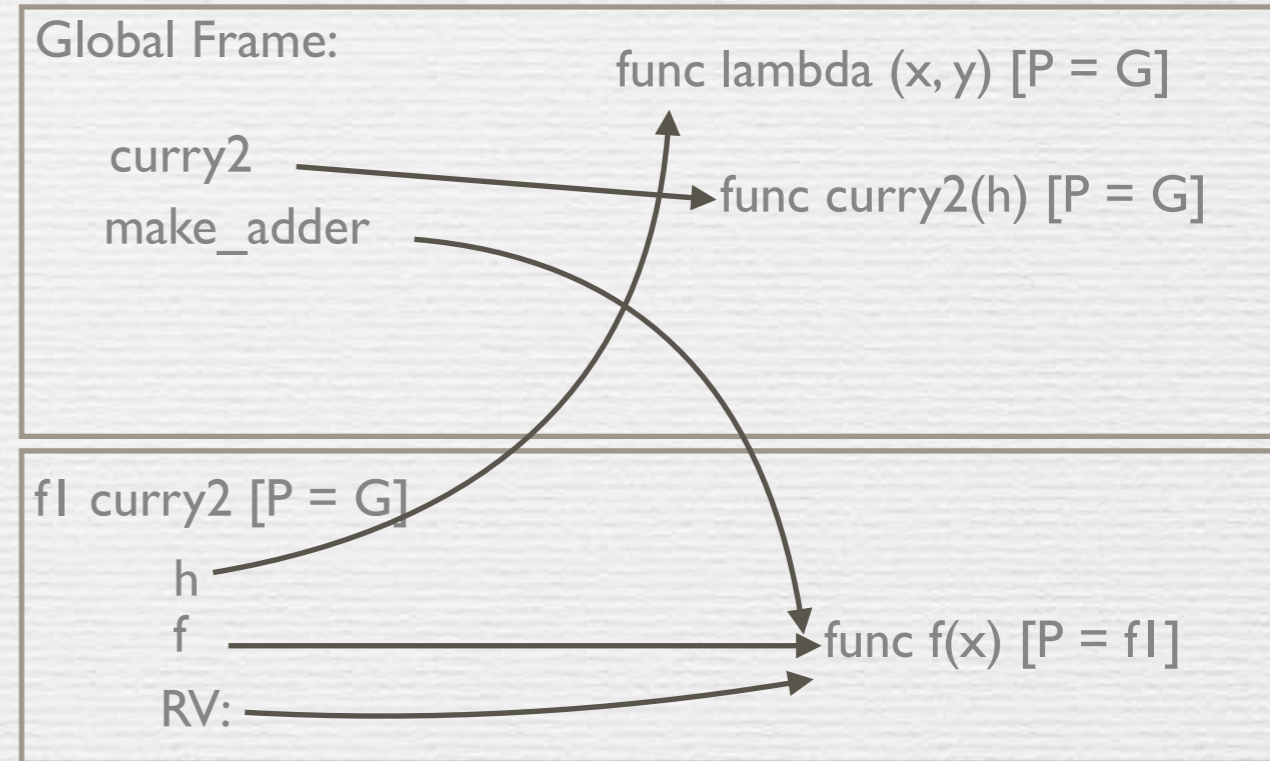
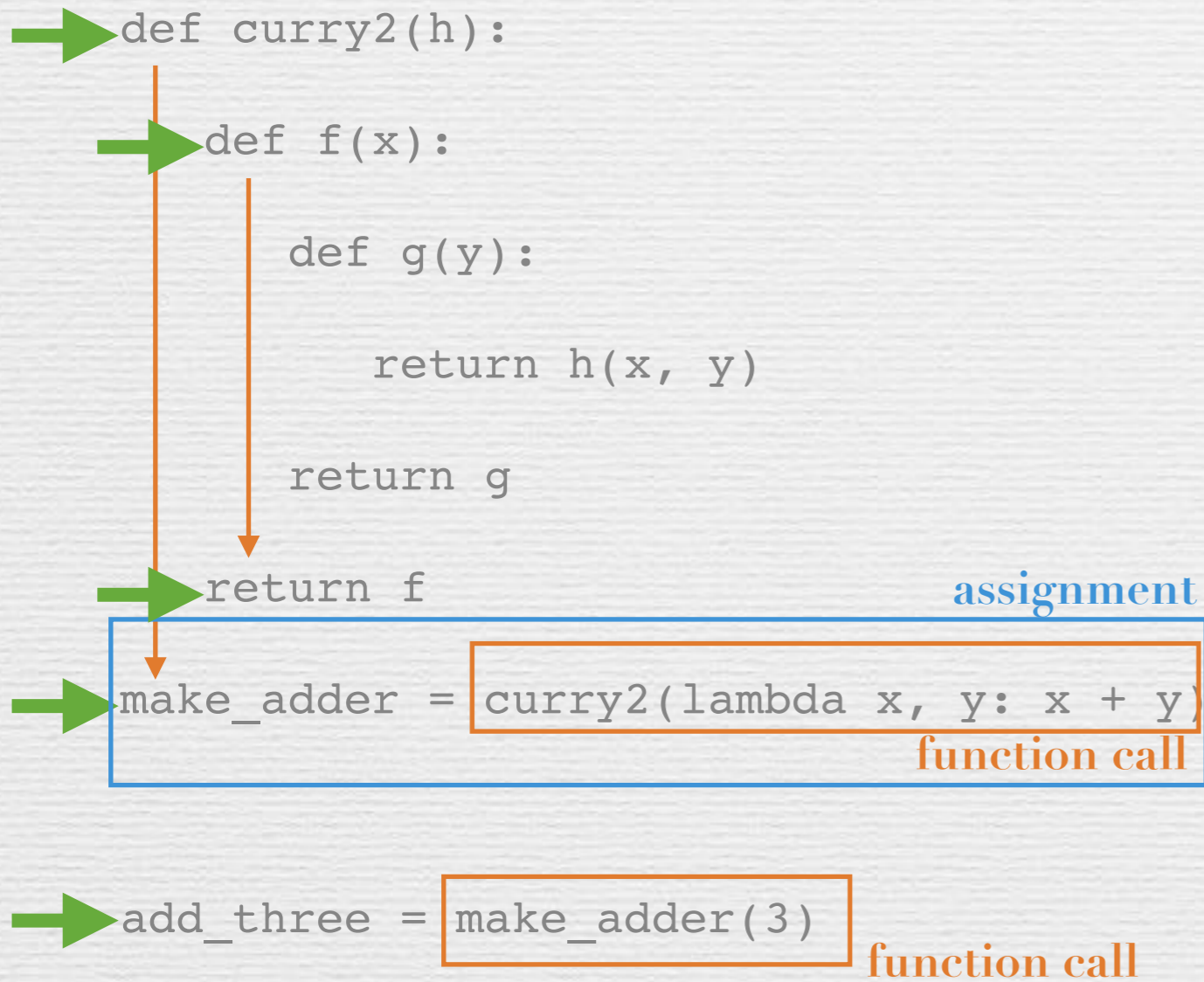
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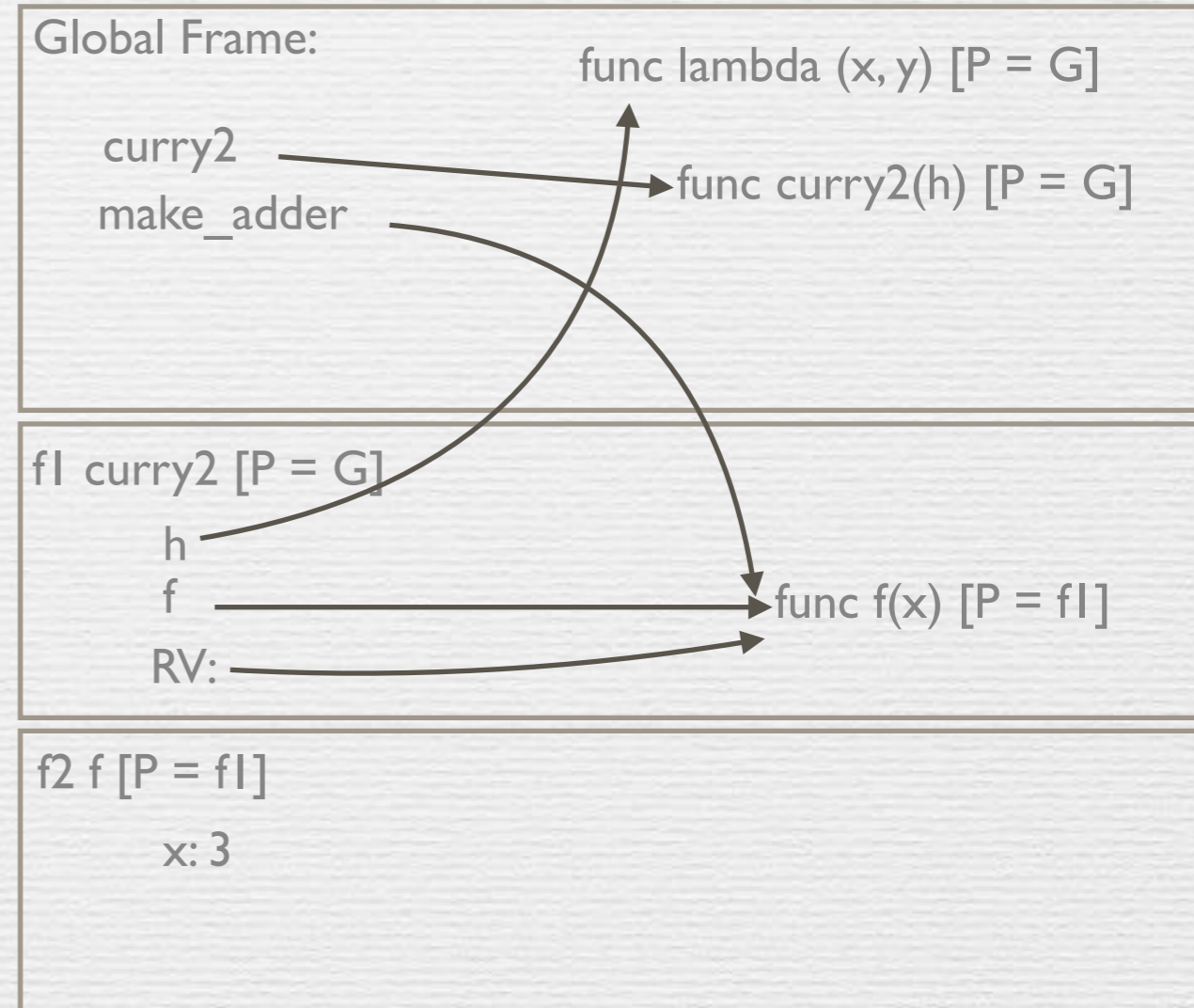
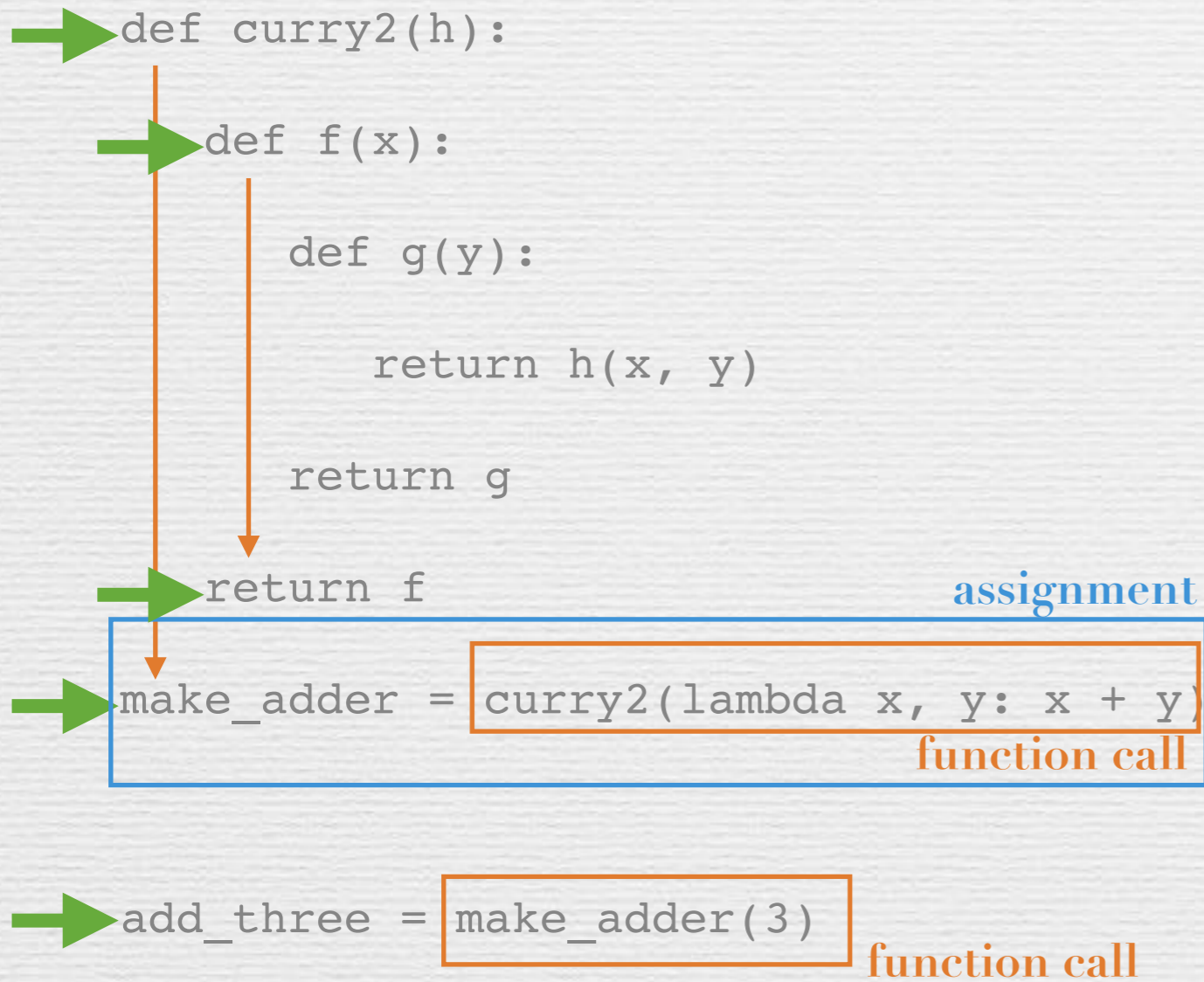
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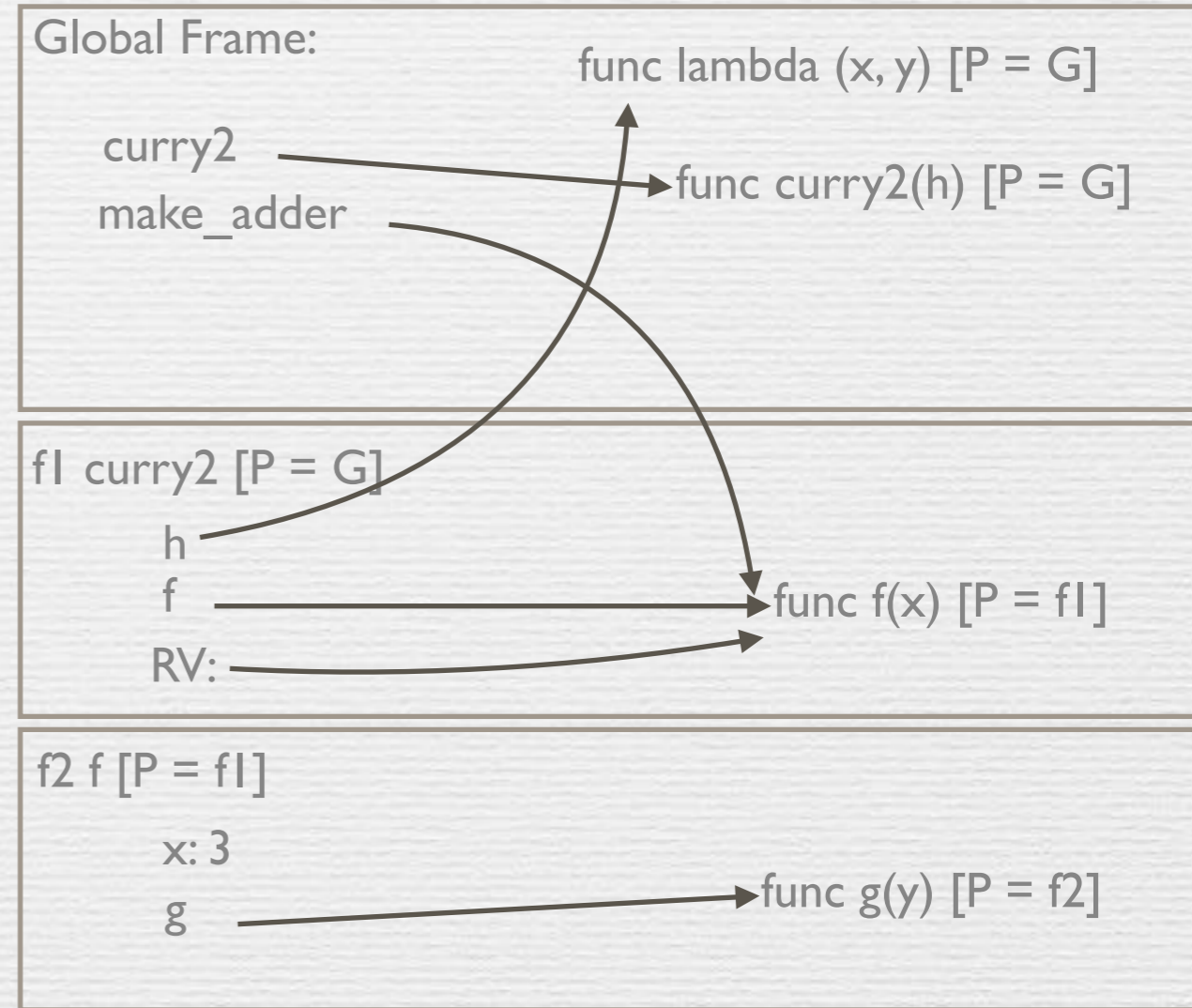
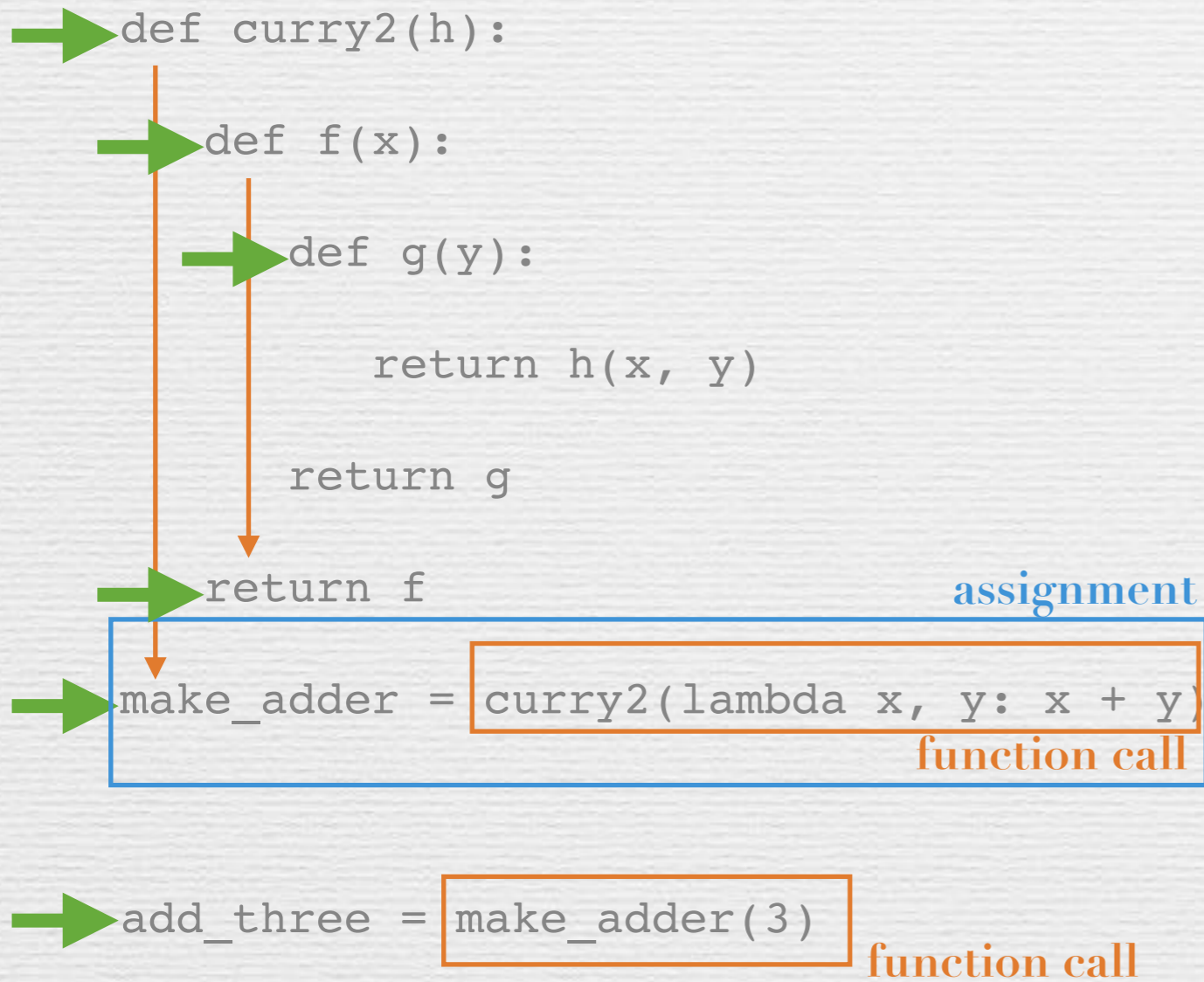
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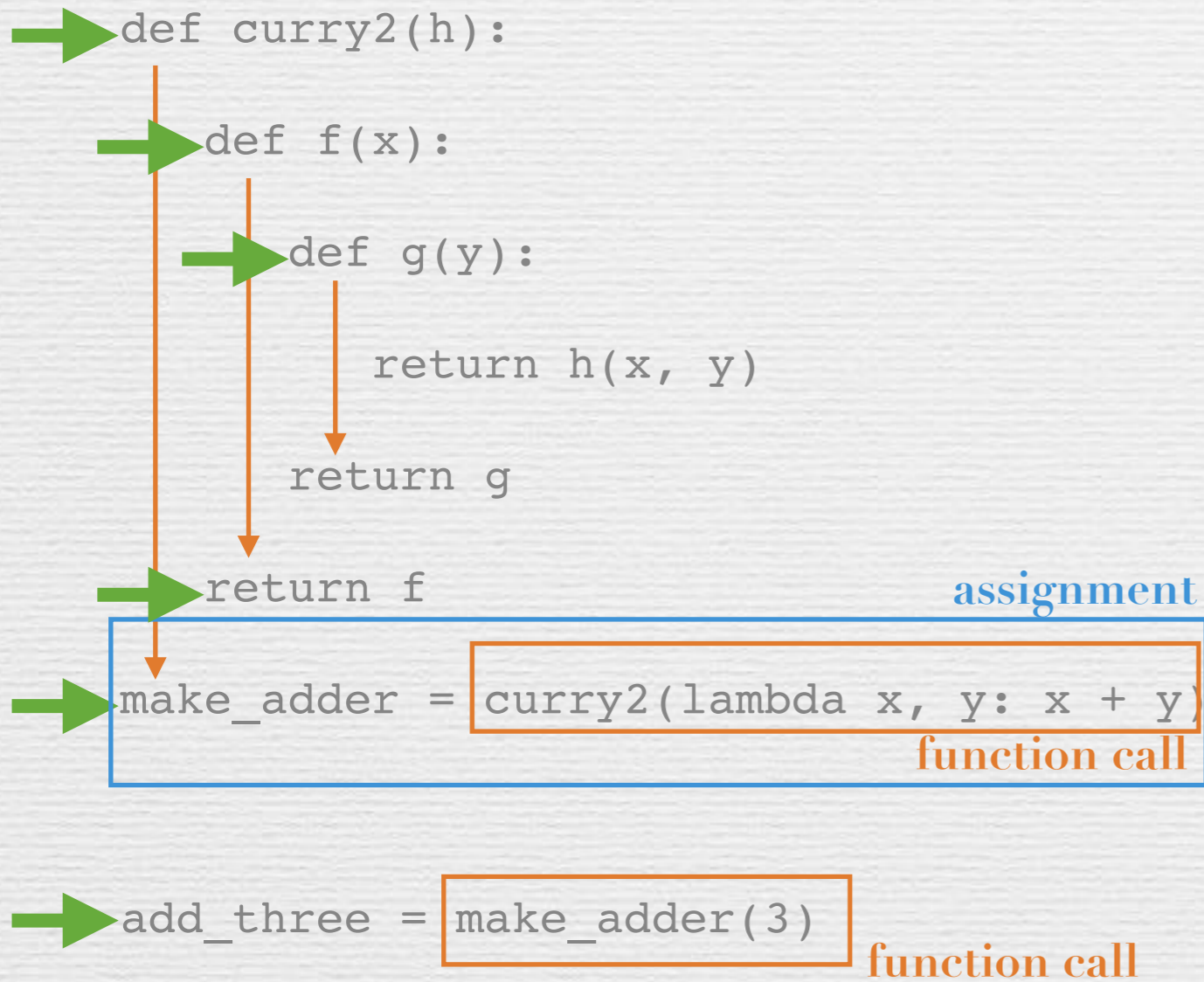
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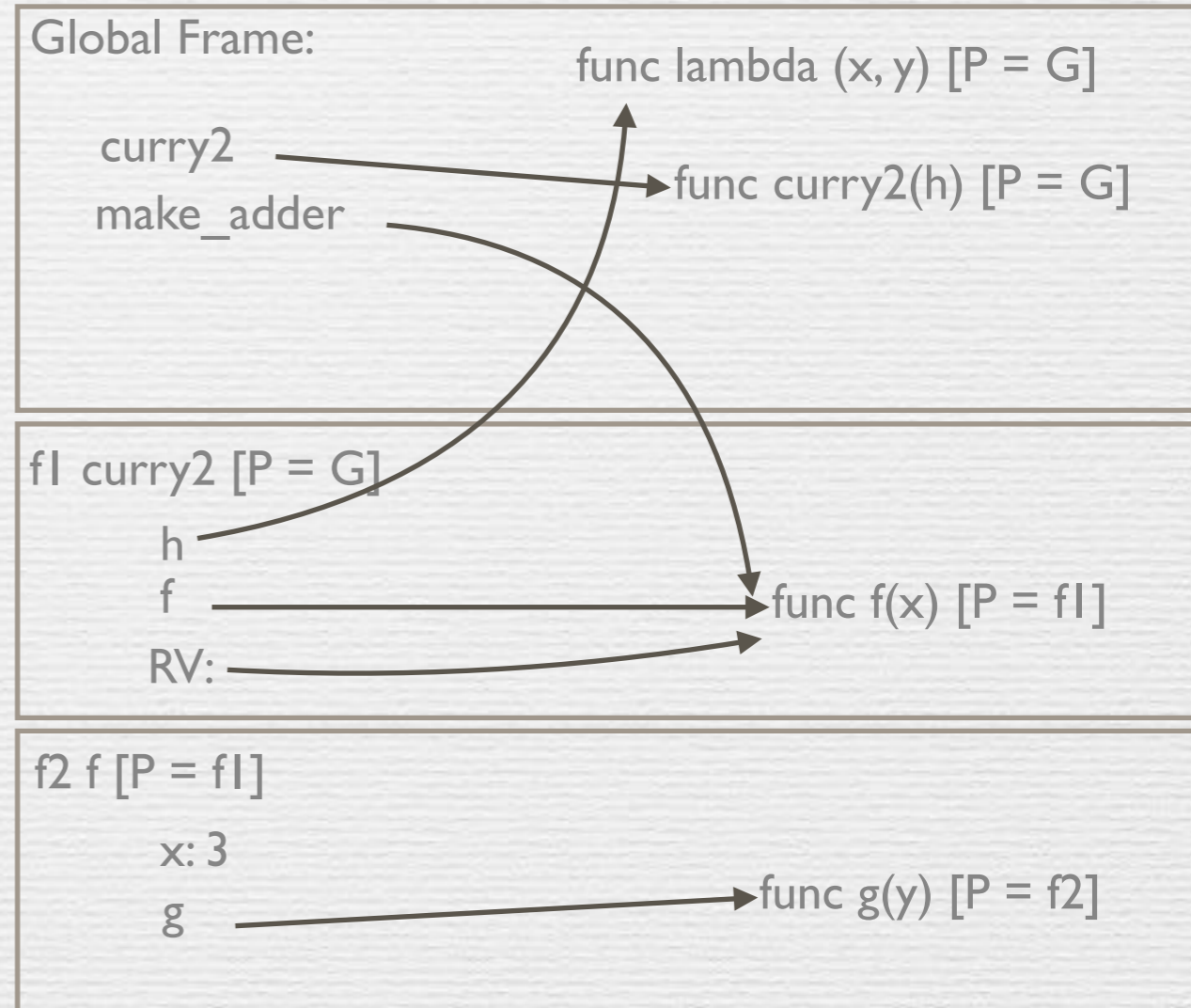


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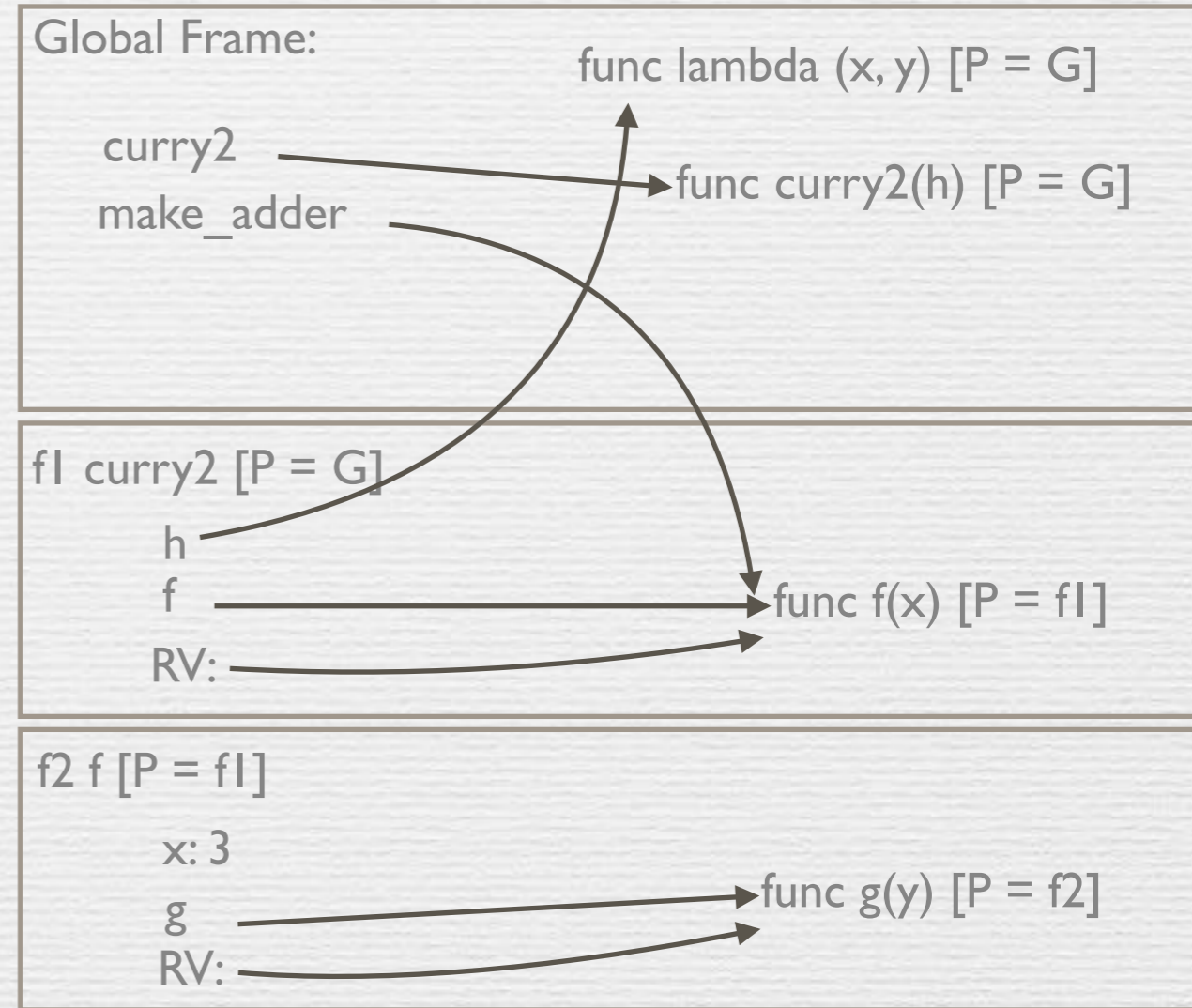
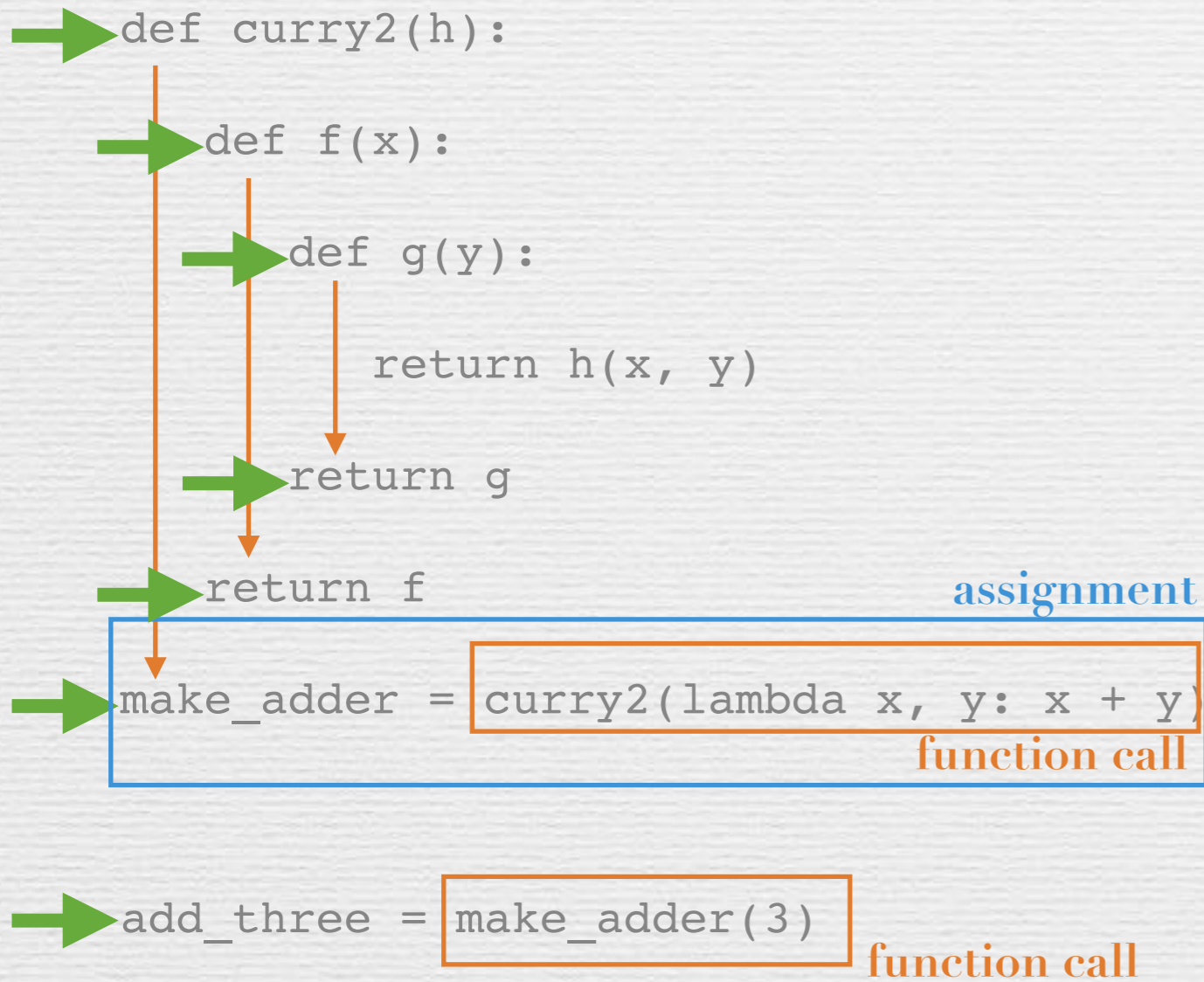
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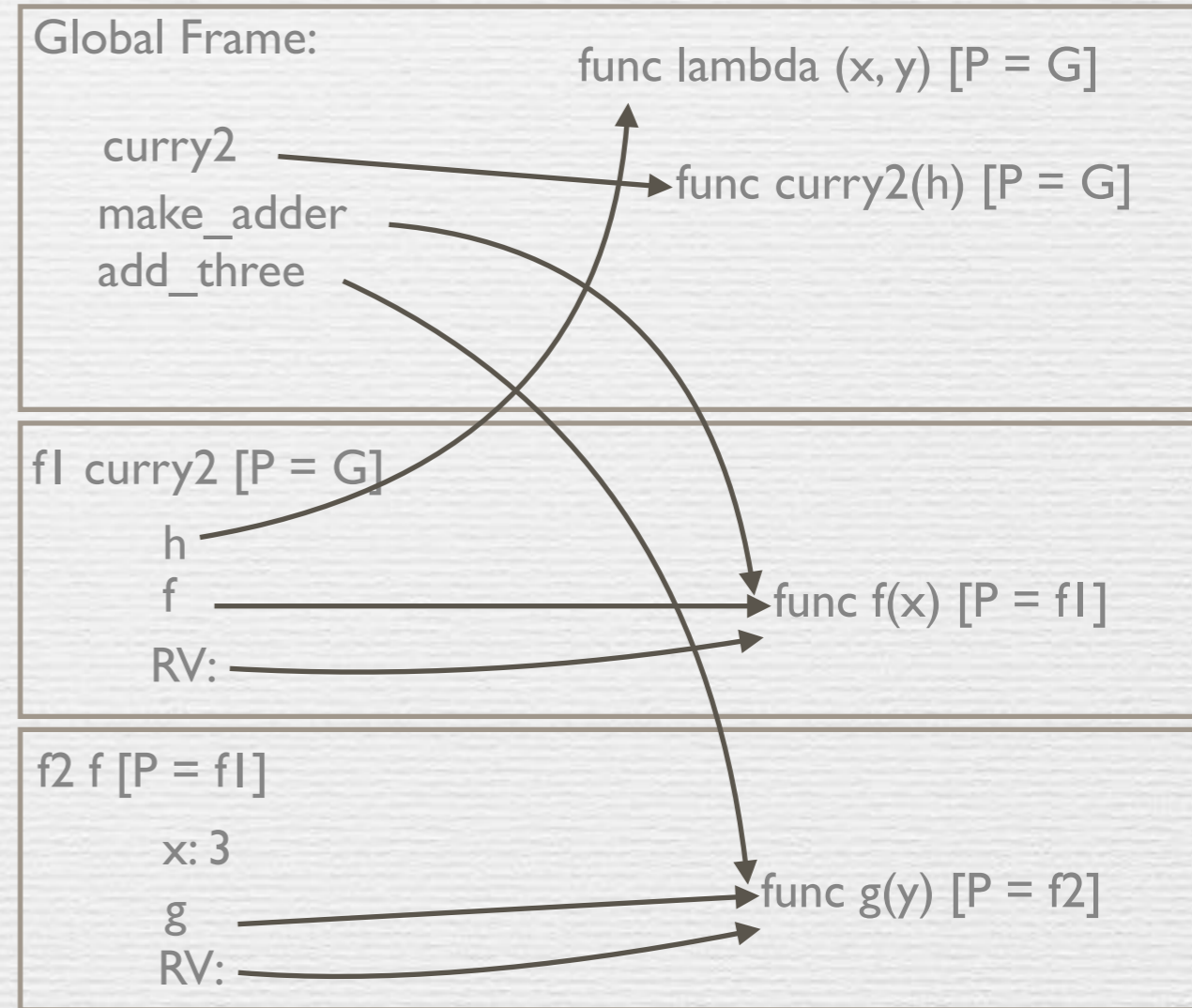
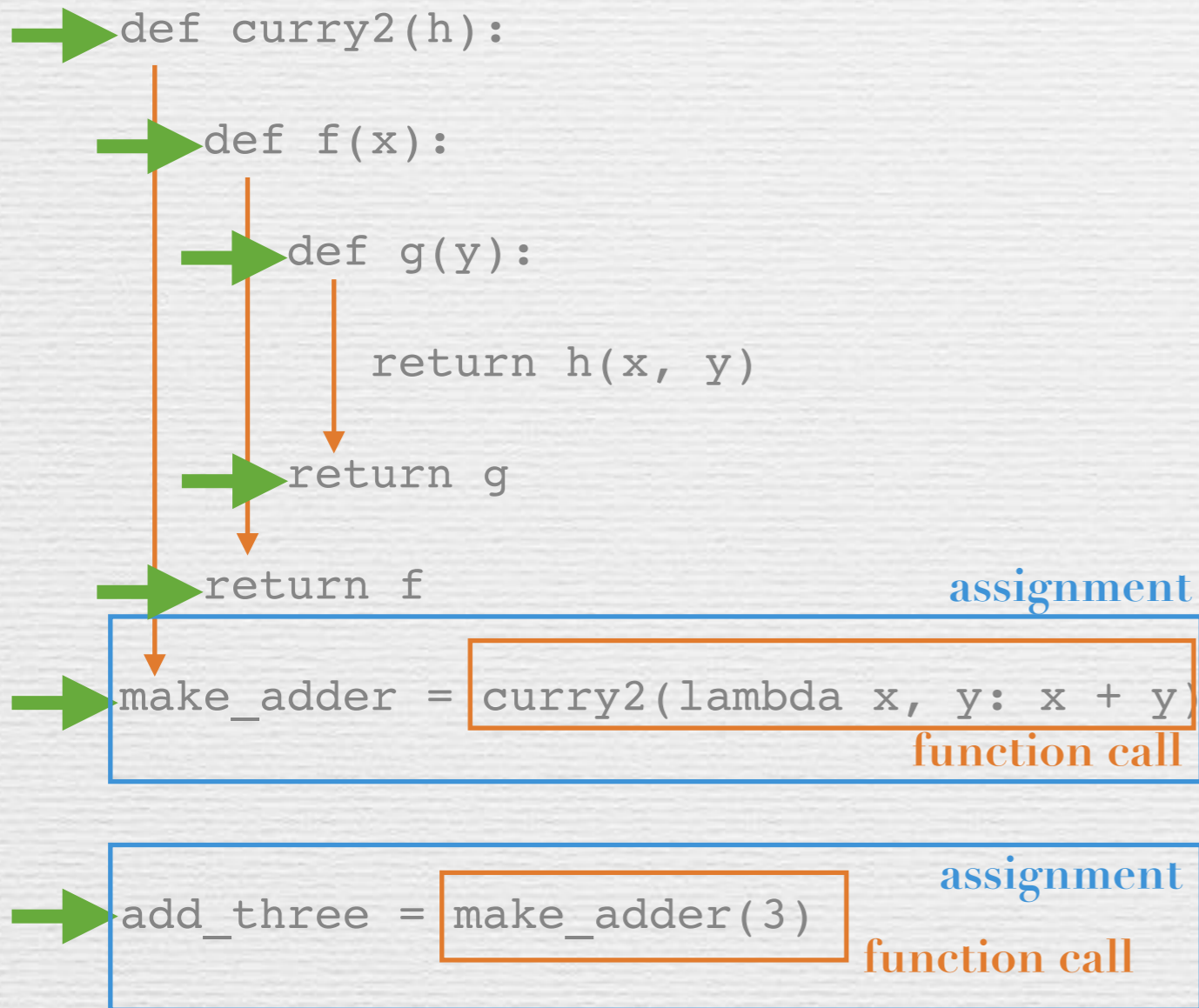


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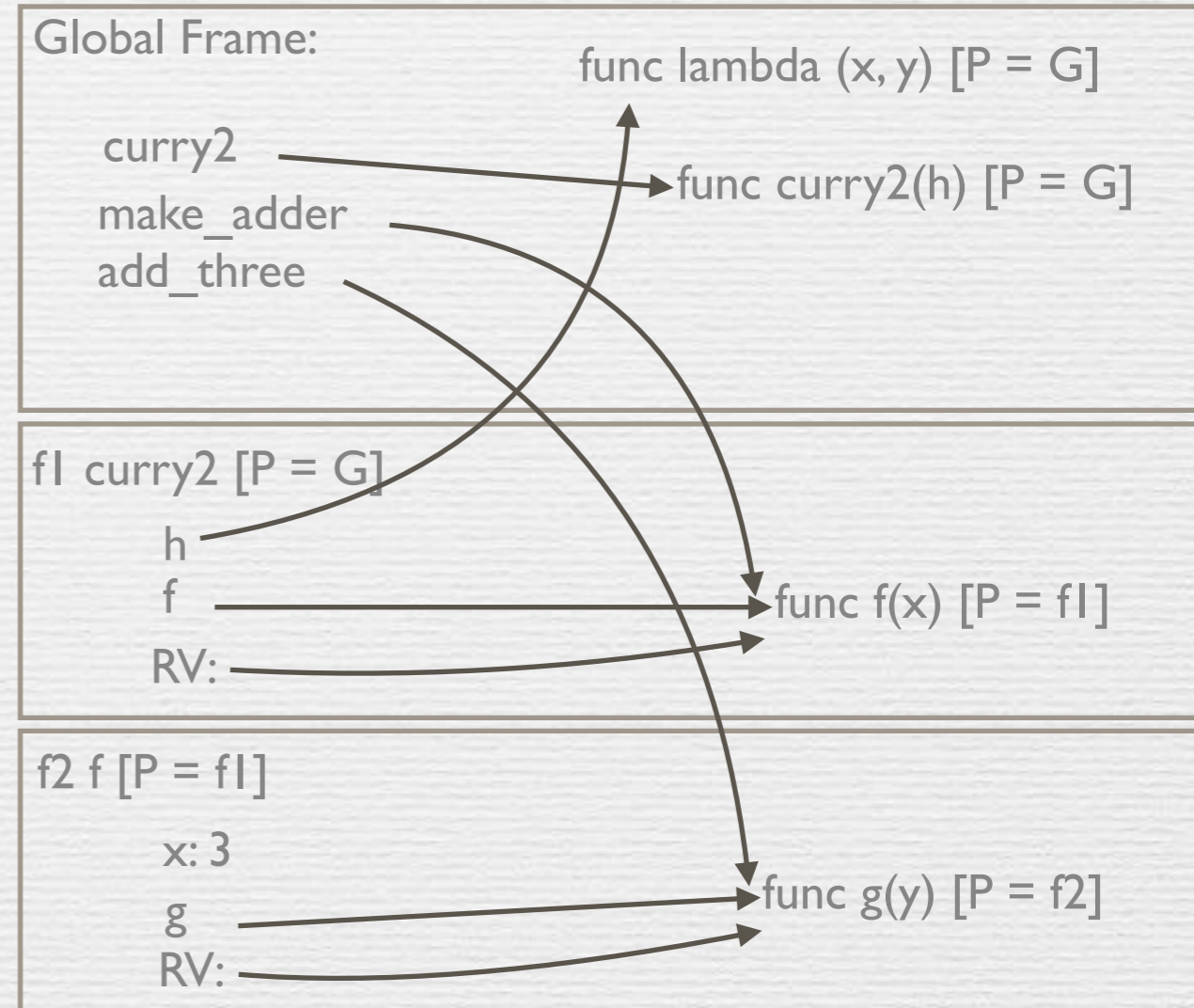
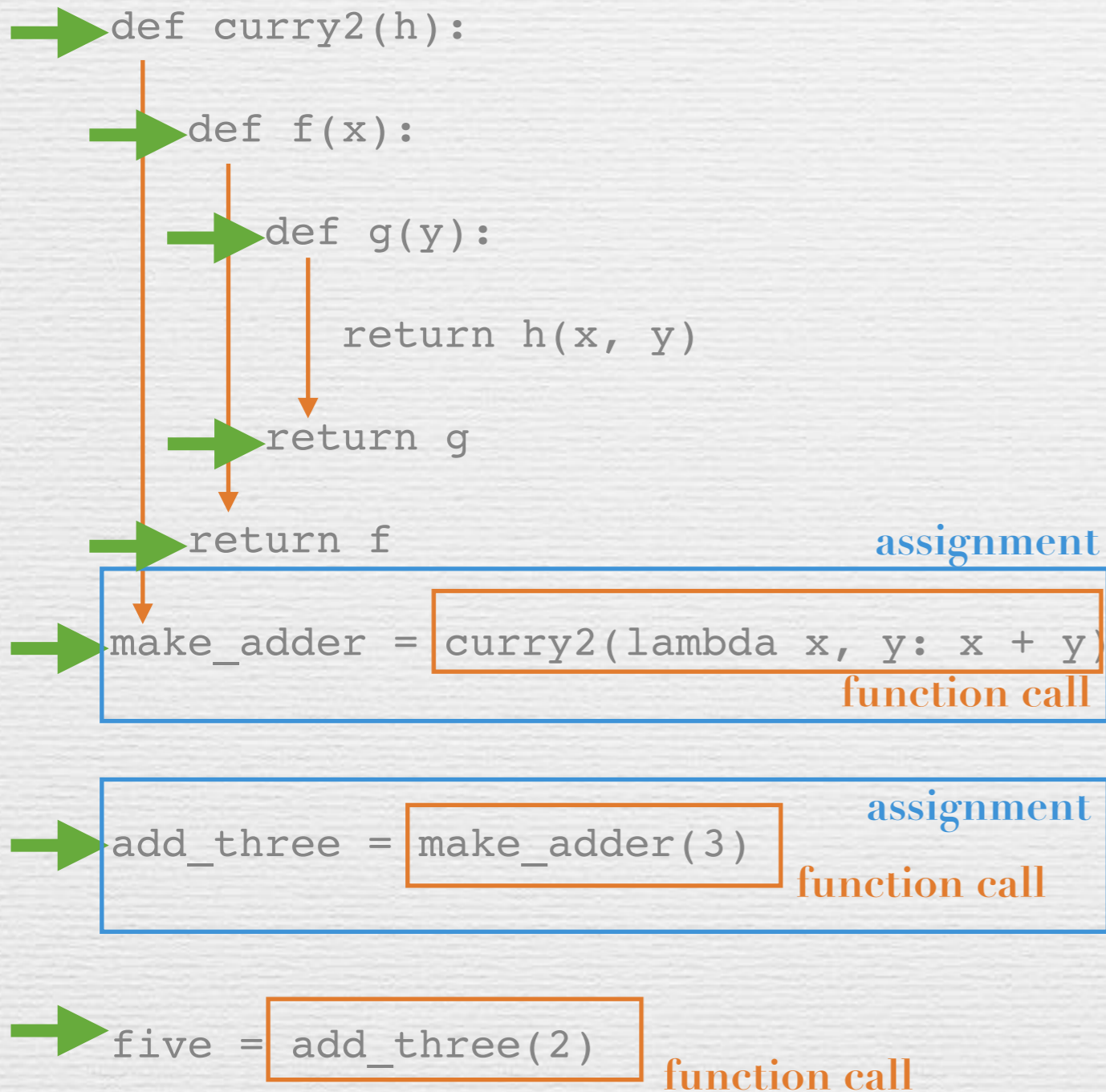
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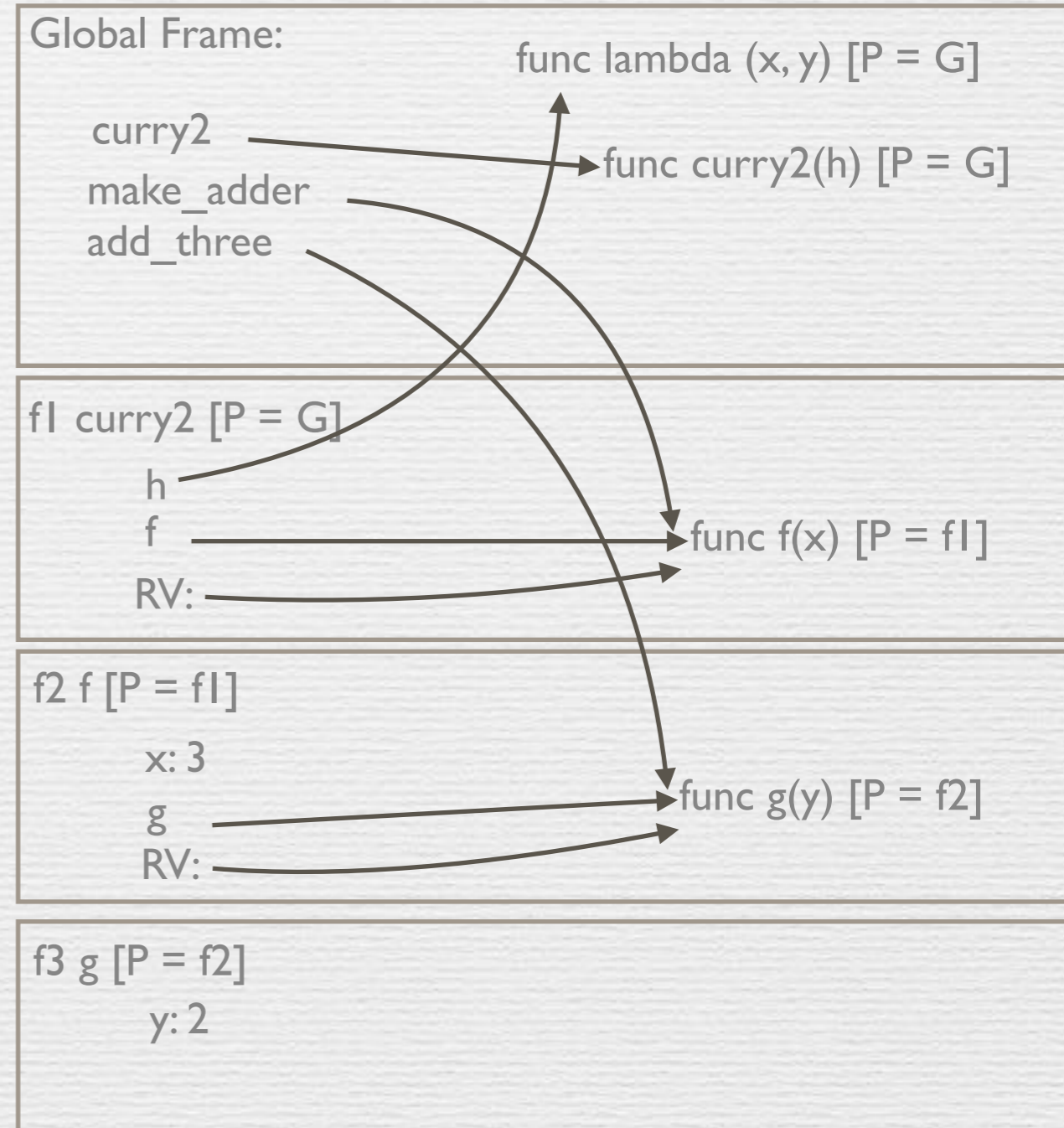
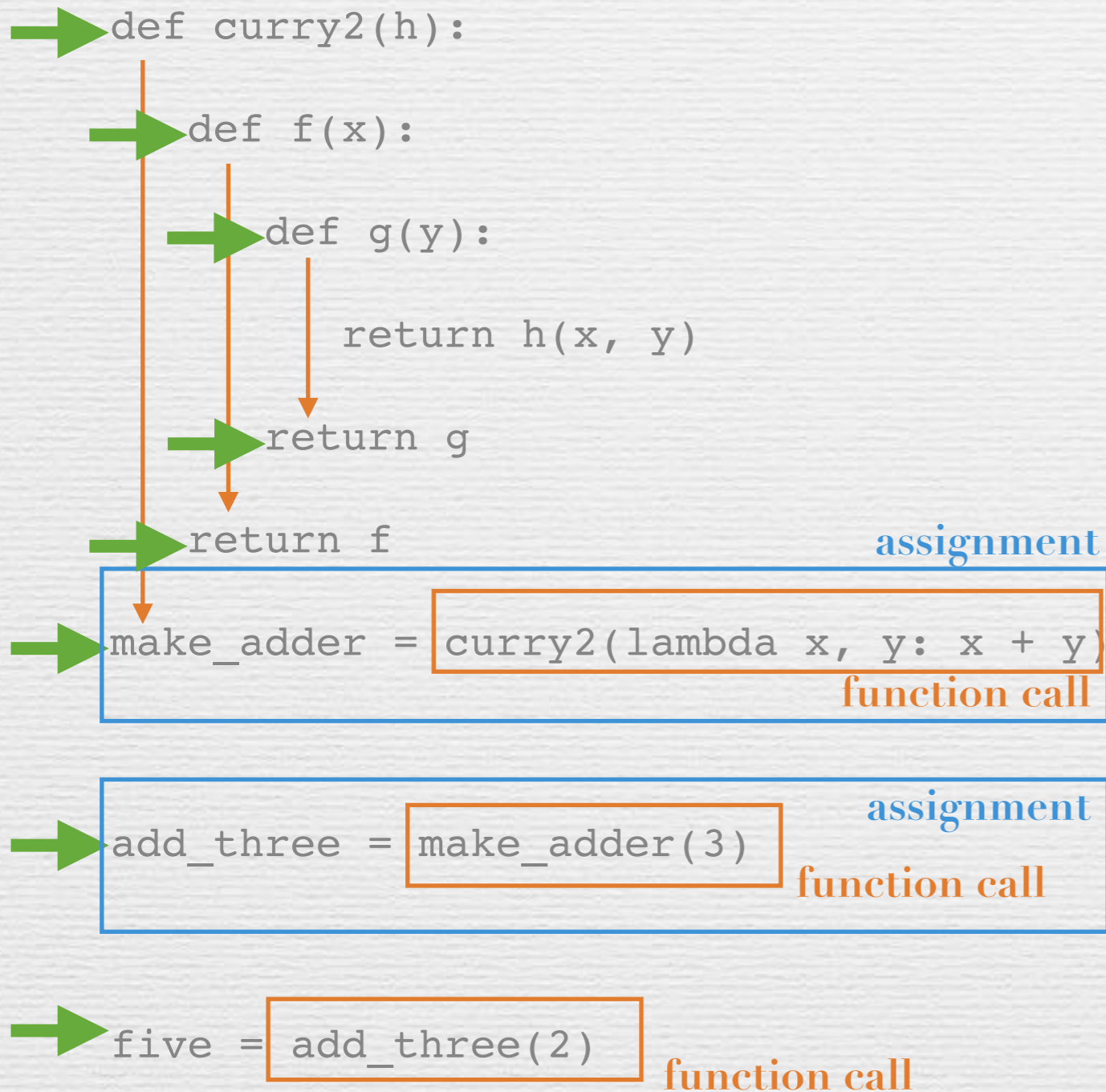
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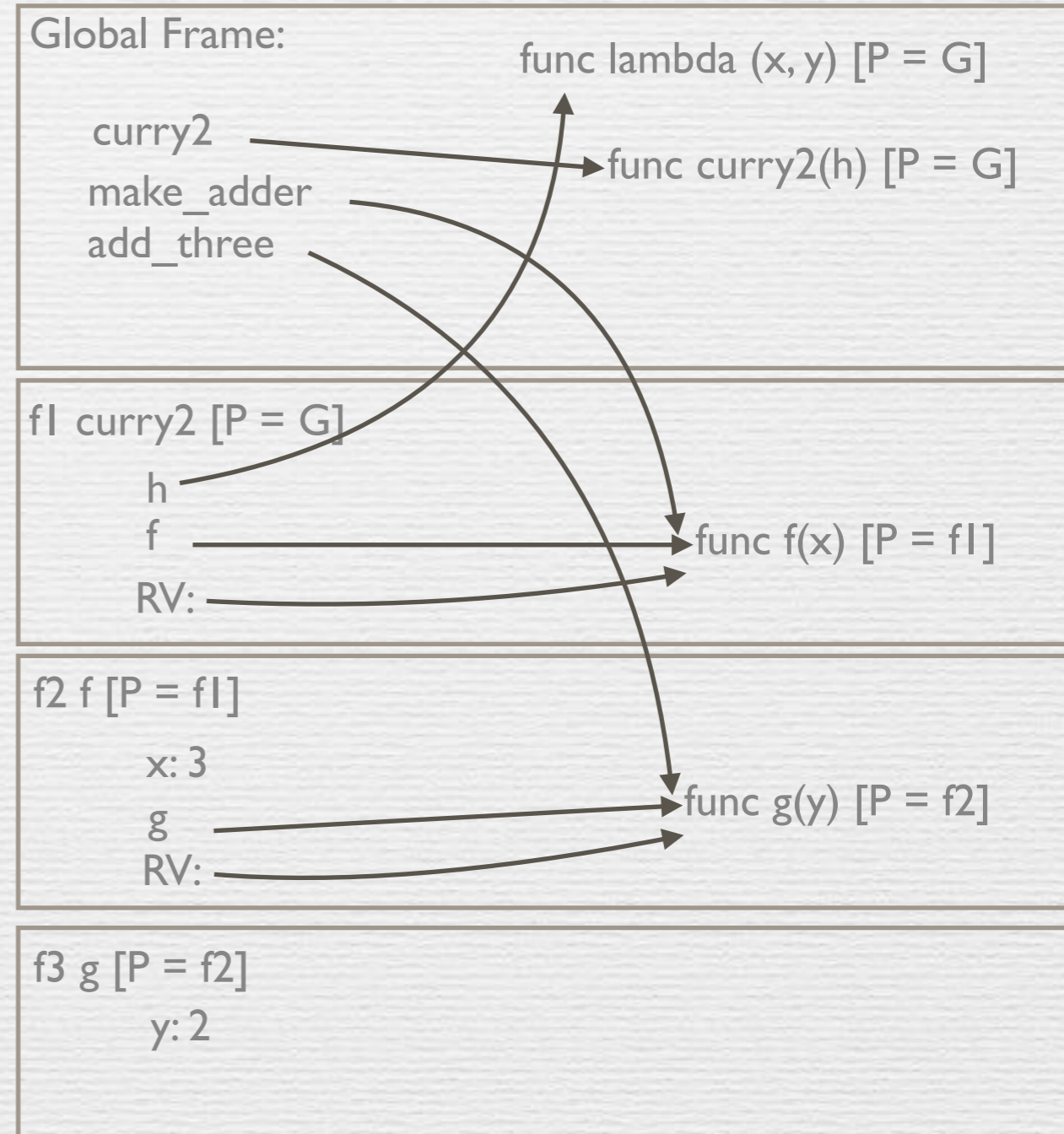
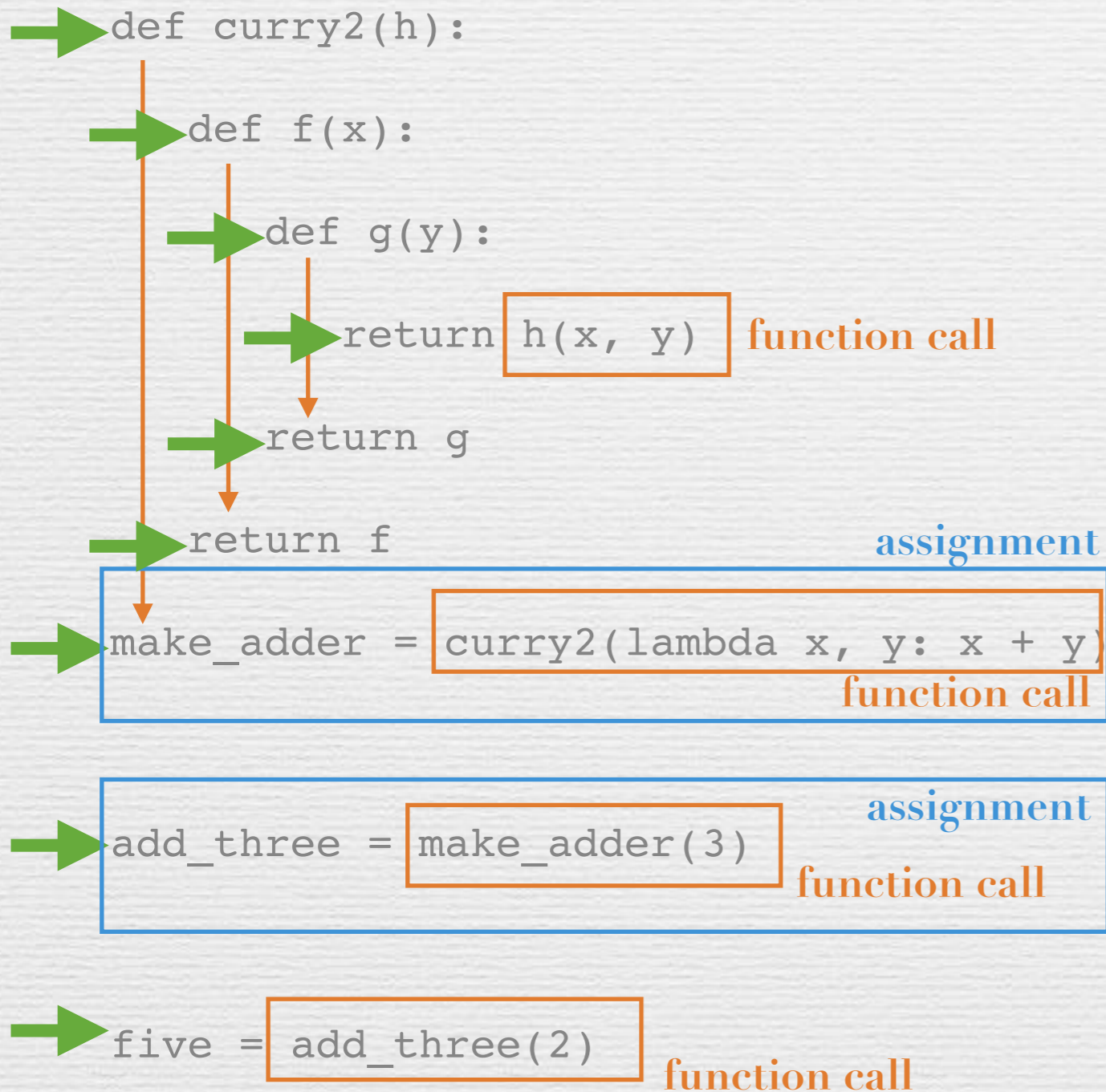
tip: when you start doing a function call, mark where you were before so that you know which line to go back to

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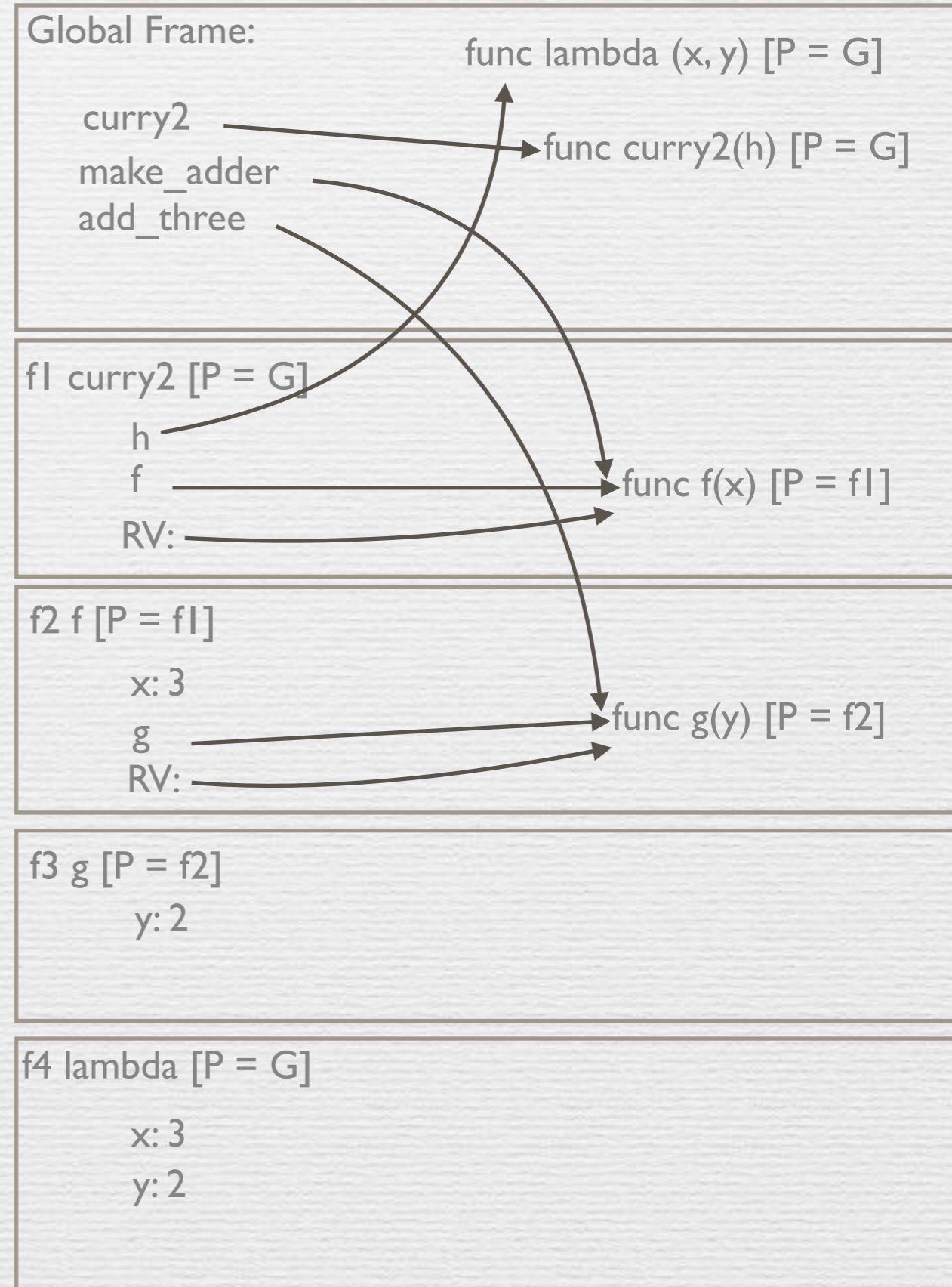
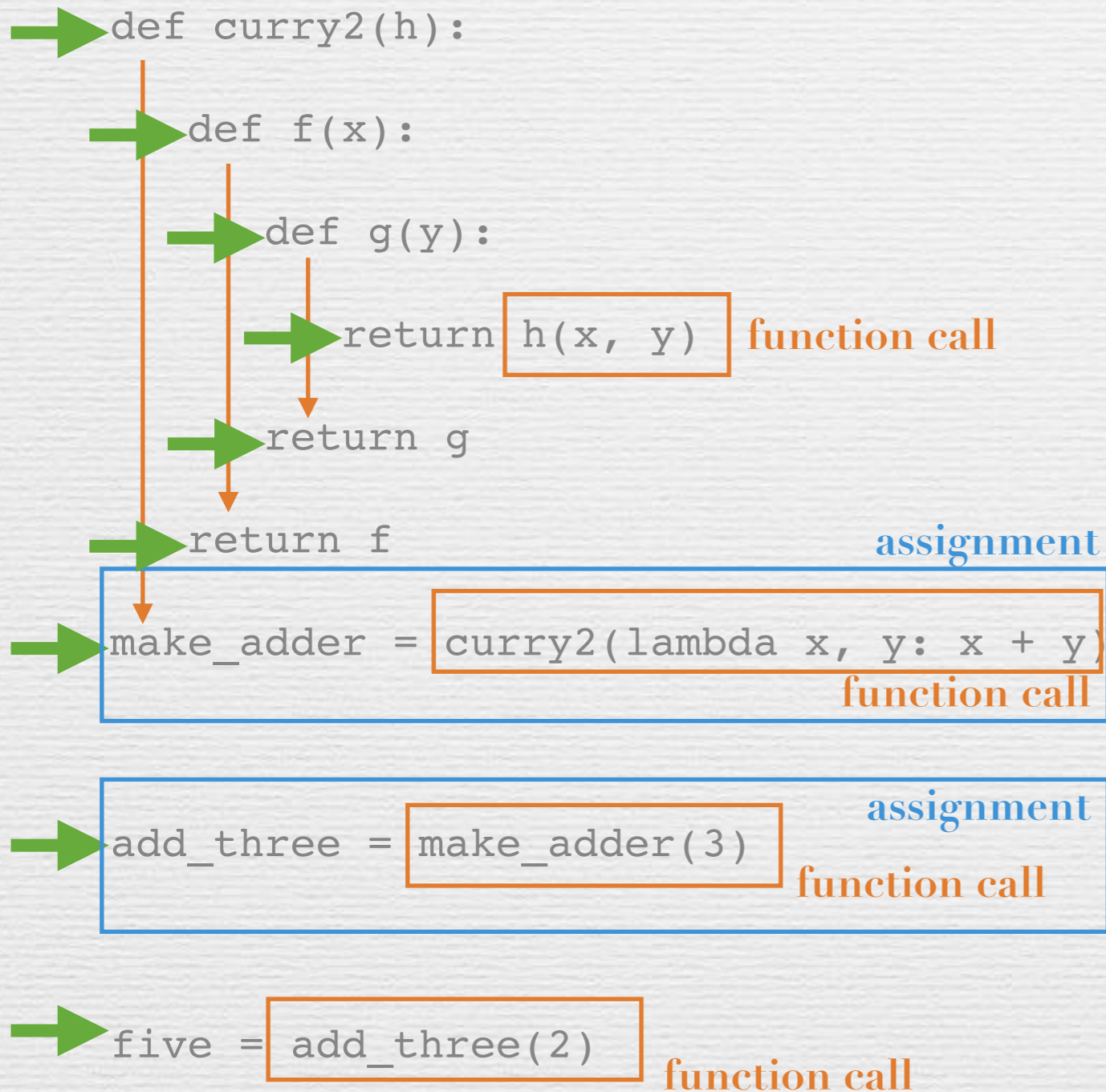
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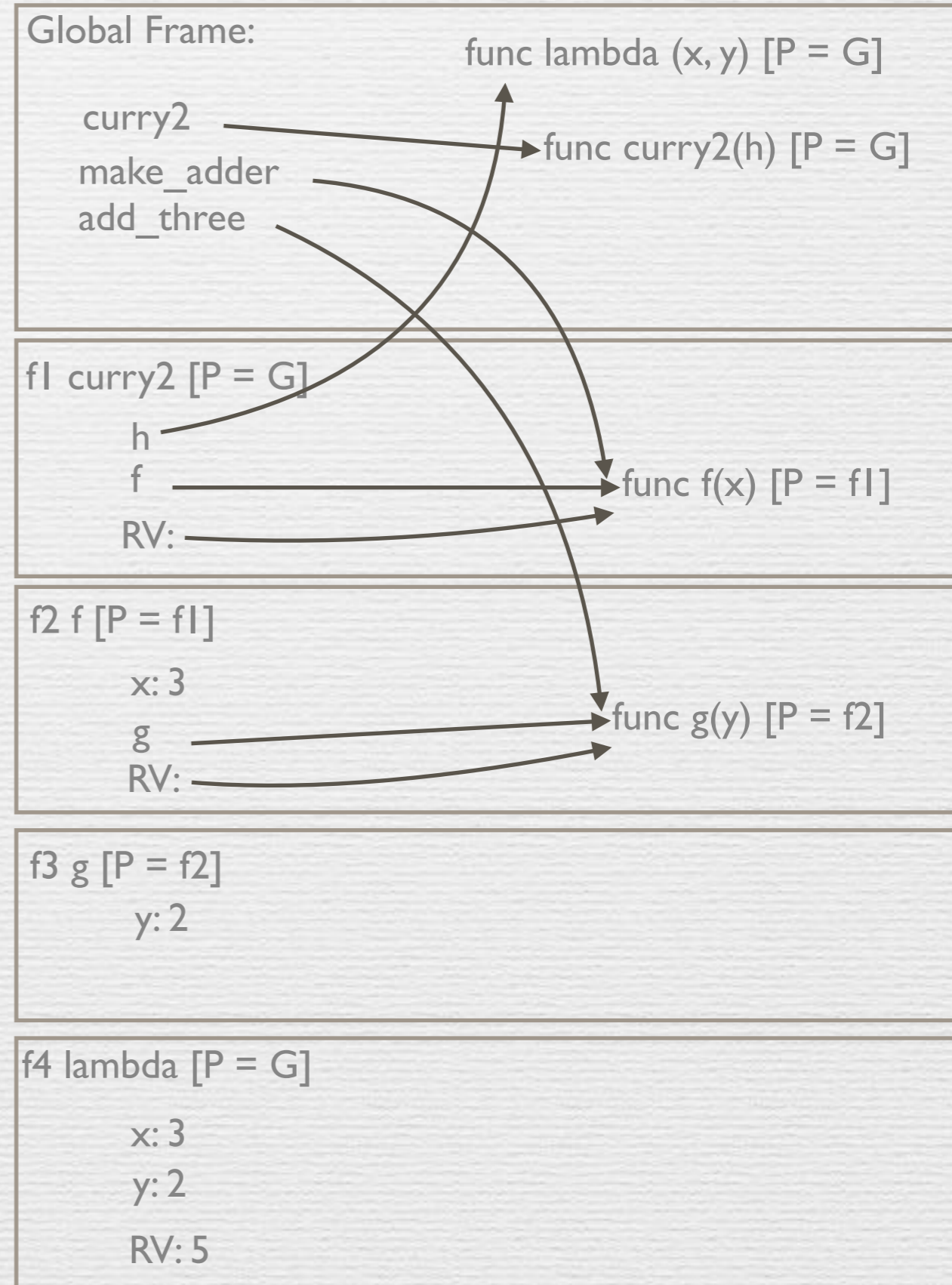
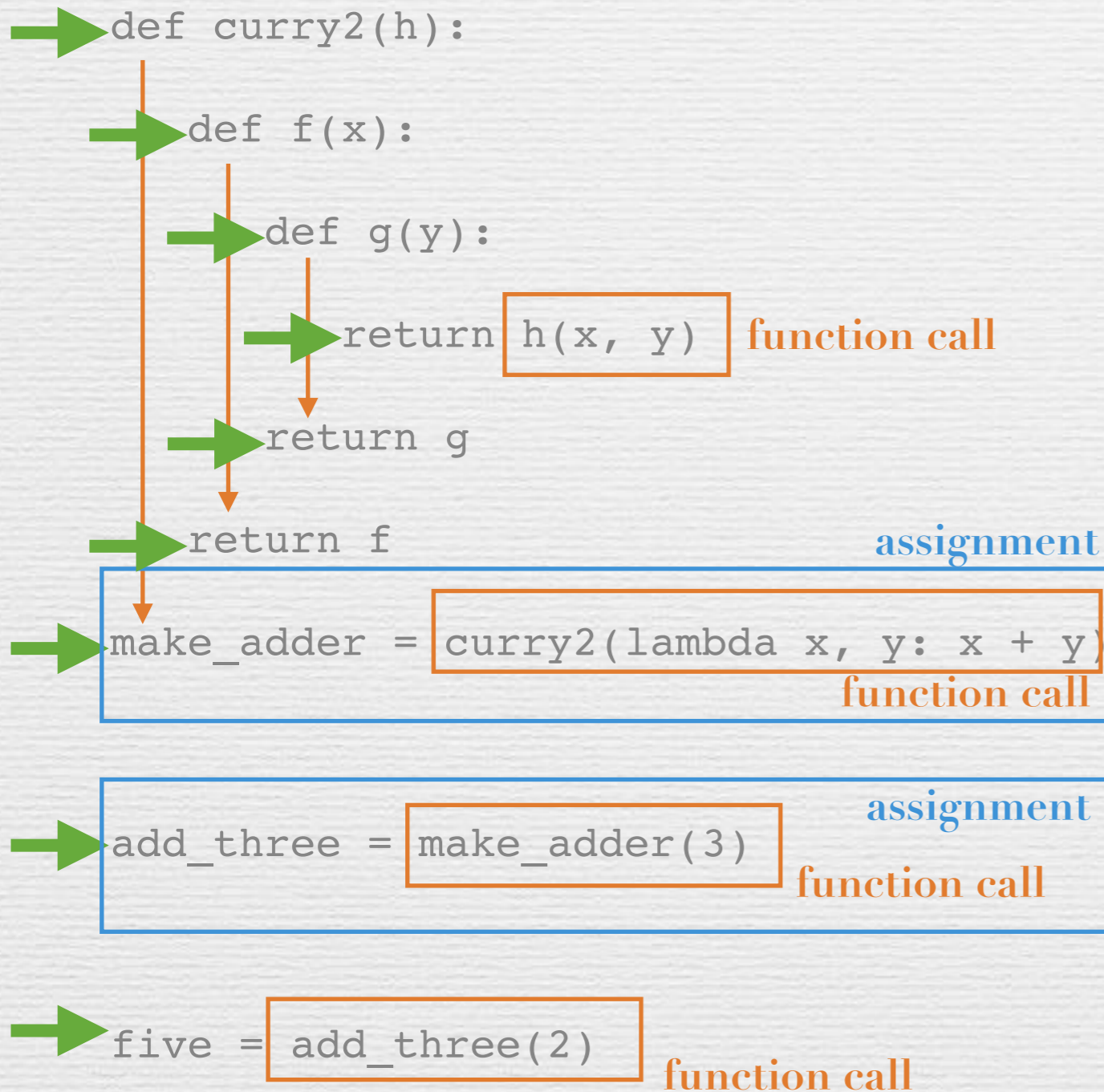
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1.5 #1



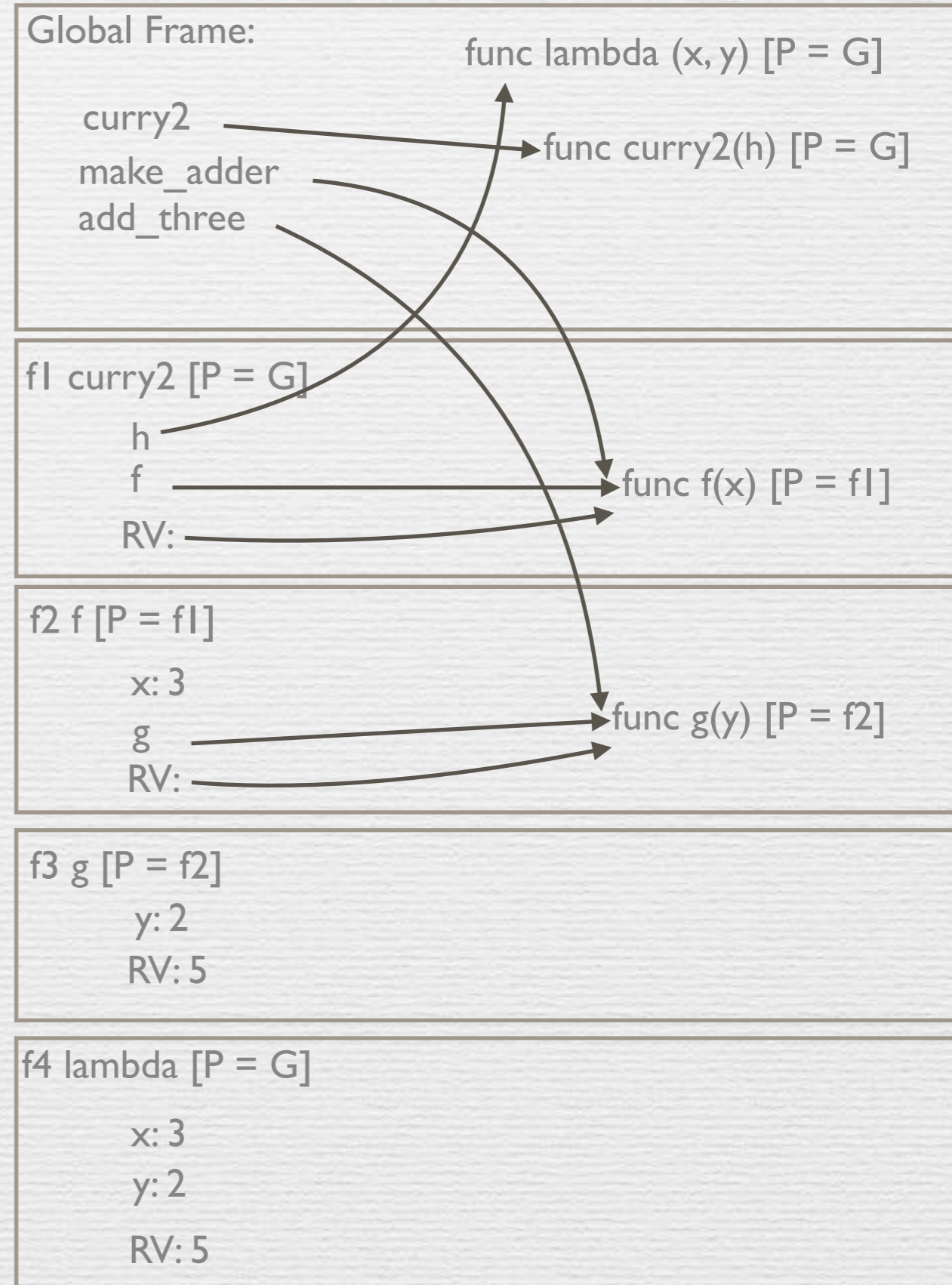
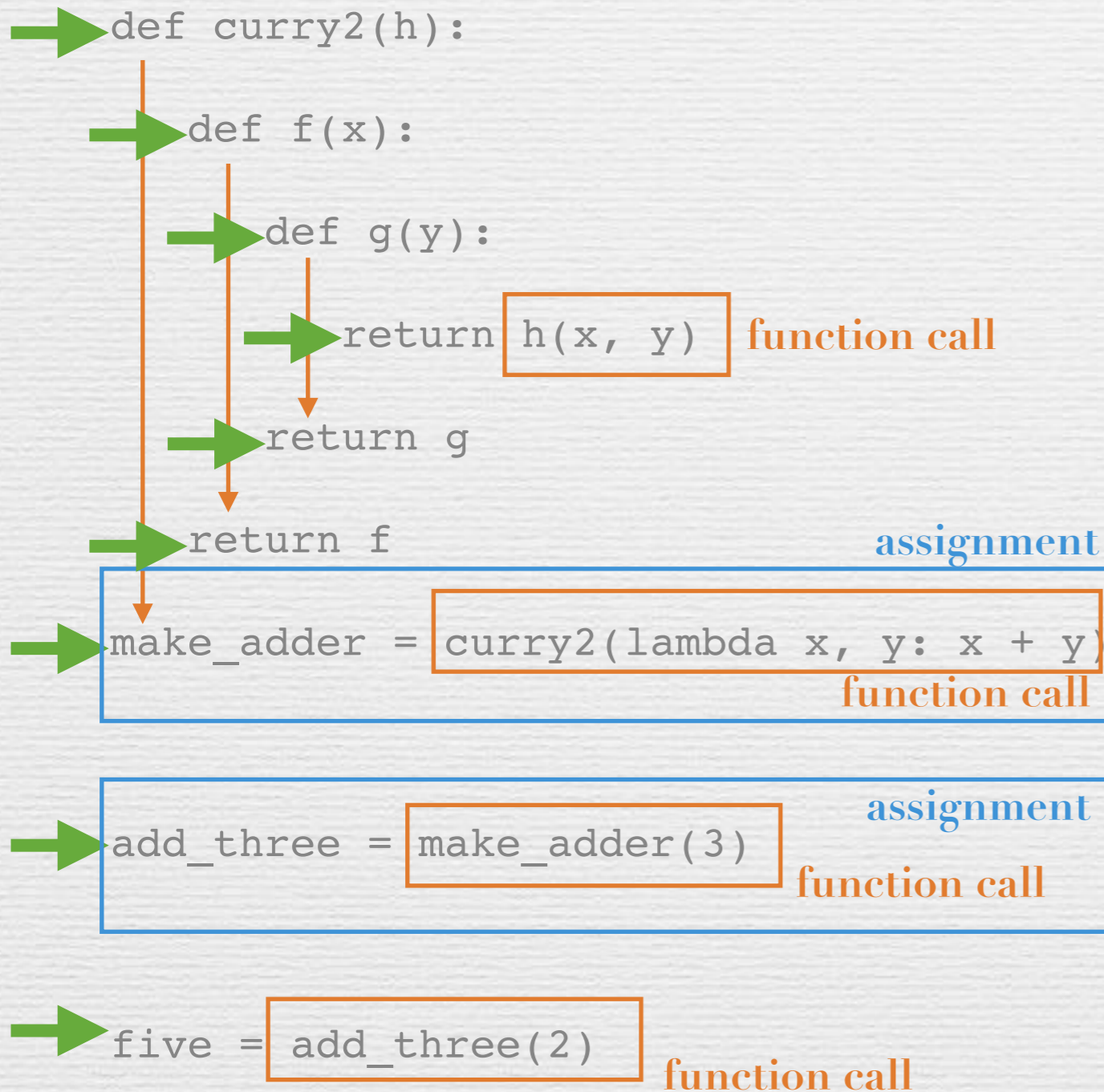
tip: when you start doing a function call, mark where you were before so that you know which line to go back to

1.5 #1



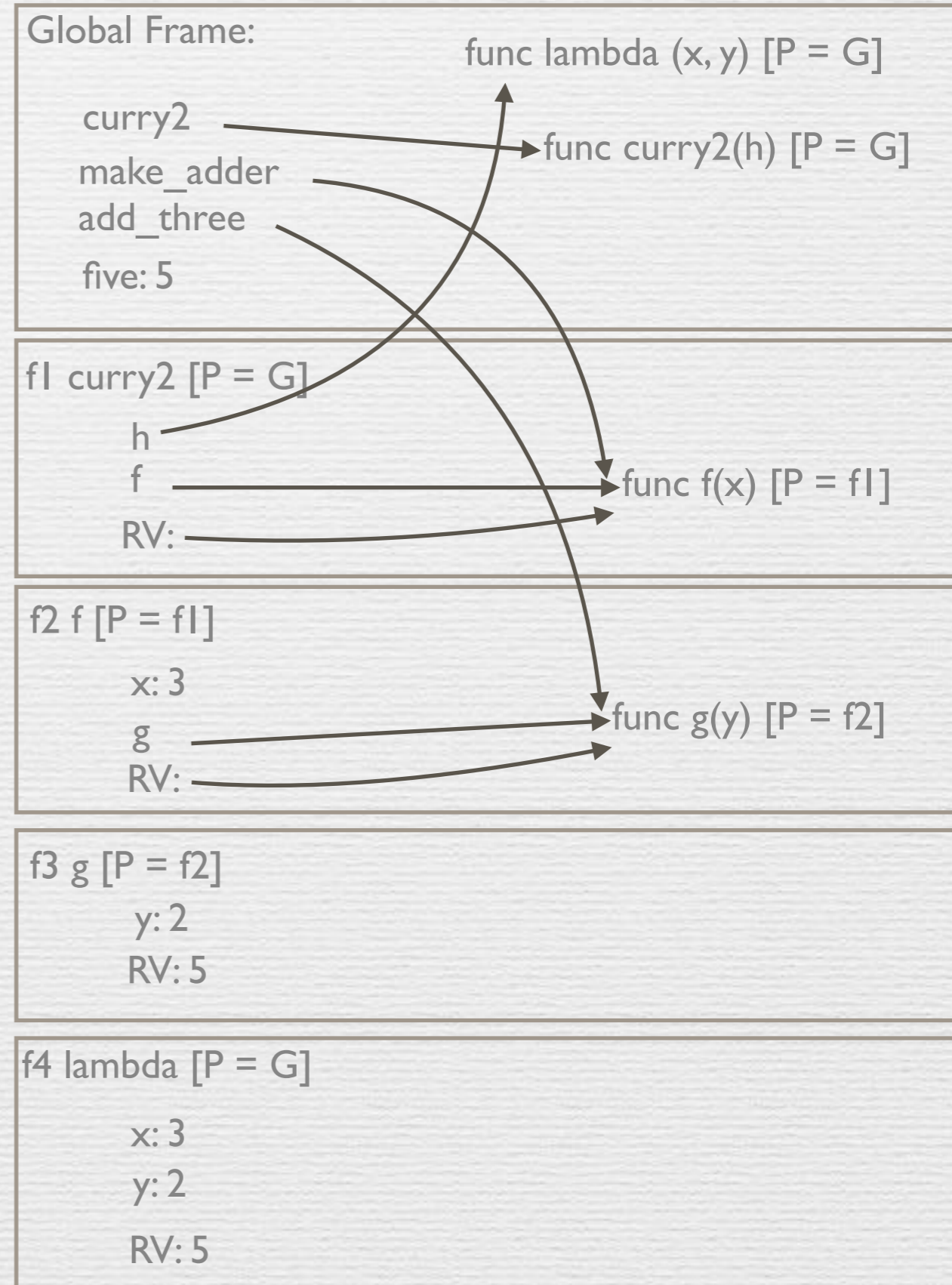
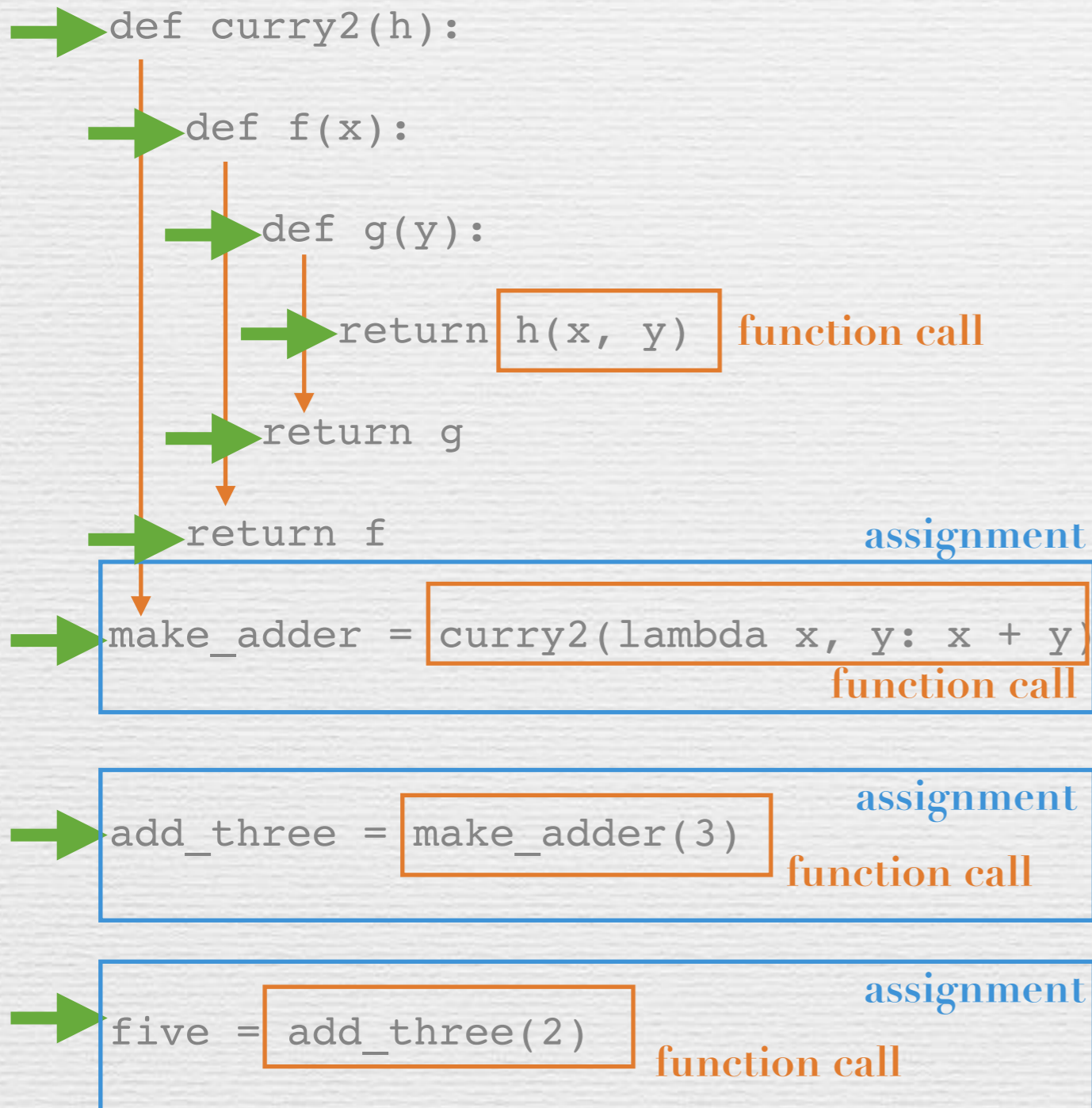
tip: when you start doing a function call, mark where you were before so that you know which line to go back to

1.5 #1



tip: when you start doing a function call, mark where you were before so that you know which line to go back to

1.5 #1



tip: when you start doing a function call, mark where you were before so that you know which line to go back to

1.5 #2

```
n = 7
def f(x):
    n = 8
    return x + 1
def g(x):
    n = 9
    def h():
        return x + 1
    return h
def f(f, x):
    return f(x + n)

f = f(g, n)

g = (lambda y: y())(f)
```

1.5 #2

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→ n = 7
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Global Frame:

n:7

1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
def g(x):
    n = 9
    def h():
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    return h
def f(f, x):
    return f(x + n)

f = f(g, n)

g = (lambda y: y())(f)
```

Global Frame:

n:7

f → func f(x) [P = G]

1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
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def g(x):
    n = 9
    def h():
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Global Frame:

n: 7

f → func f(x) [P = G]

1.5 #2

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→ n = 7
→ def f(x):
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    return f(x + n)

f = f(g, n)

g = (lambda y: y())(f)
```

Global Frame:

```
n: 7
f → func f(x) [P = G]
g → func g(x) [P = G]
```

1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
def f(f, x):
    return f(x + n)
```

```
f = f(g, n)
```

```
g = (lambda y: y())(f)
```

Global Frame:

n:7

f → func f(x) [P = G]

g → func g(x) [P = G]

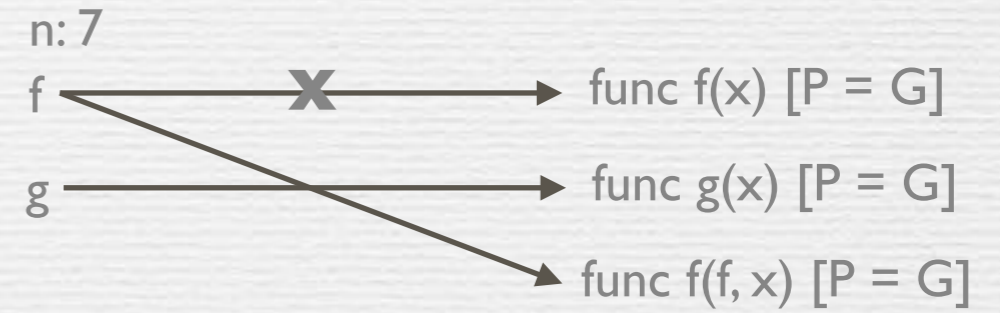
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    return f(x + n)
```

```
f = f(g, n)
```

```
g = (lambda y: y())(f)
```

Global Frame:



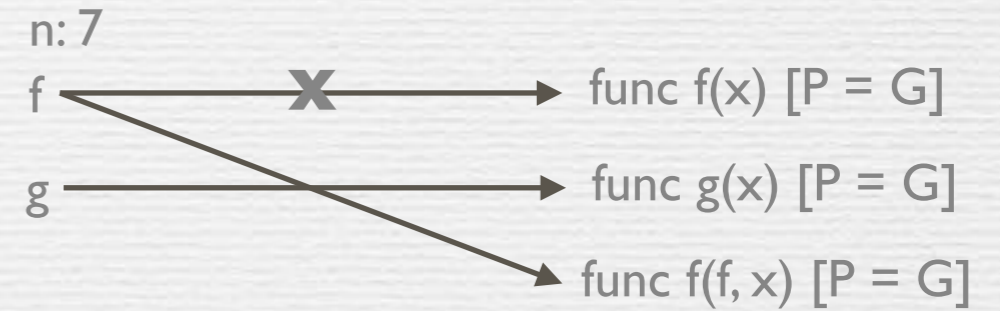
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    return f(x + n)
```

```
f = f(g, n)
```

```
g = (lambda y: y())(f)
```

Global Frame:



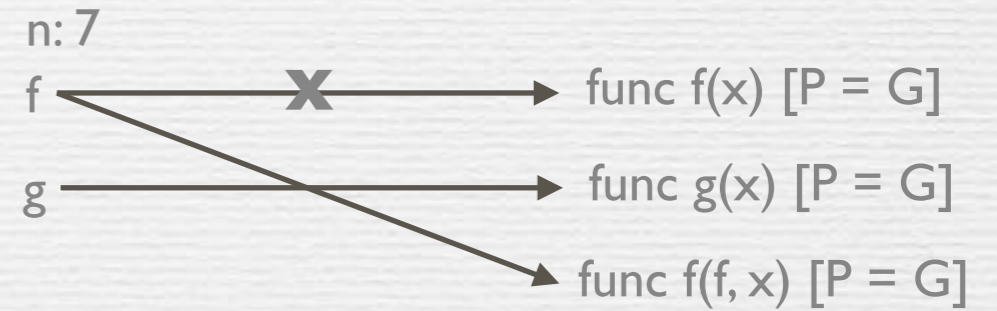
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    return f(x + n)
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



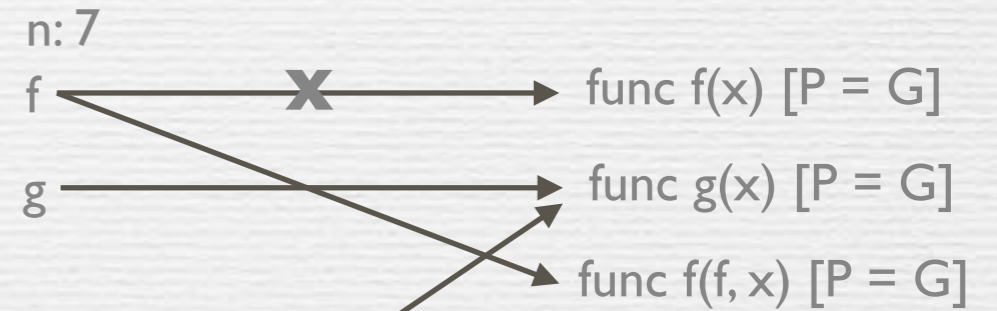
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    return f(x + n)
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]

f
x: 7

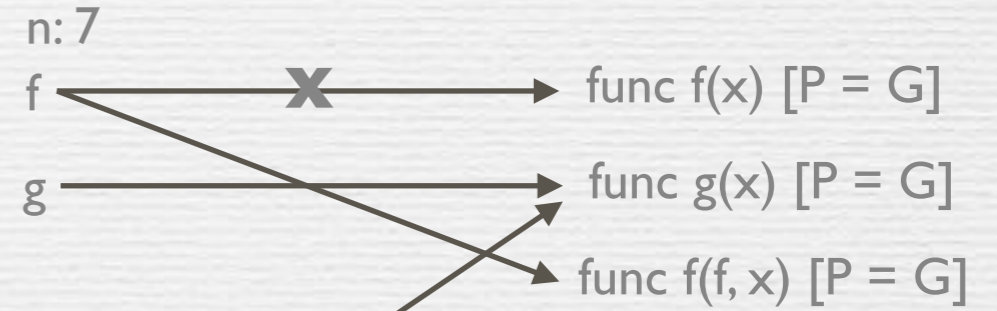
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]

f
x: 7

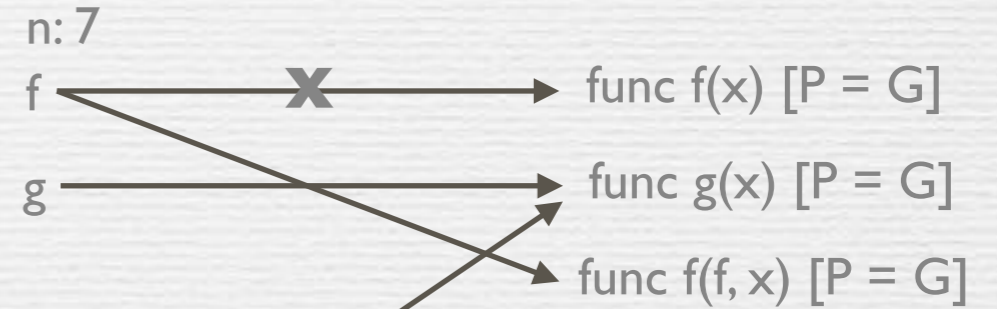
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]
f
x: 7

f2: g [P = G]
x: 14

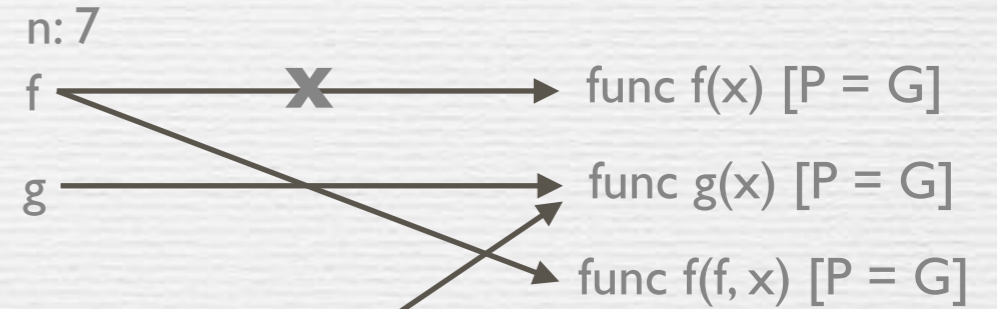
1.5 #2

```
→ n = 7
→ def f(x):
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    return x + 1
→ def g(x):
    → n = 9
    def h():
        return x + 1
    return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]
f
x: 7

f2: g [P = G]
x: 14
n: 9

1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    → n = 9
    → def h():
        return x + 1
    return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:

```
n: 7
f → func f(x) [P = G]
g → func g(x) [P = G]
    → func f(f, x) [P = G]
```

```
f1: f [P = G]
    f
    x: 7
```

```
f2: g [P = G]
    x: 14
    n: 9
    h → func h() [P = f2]
```

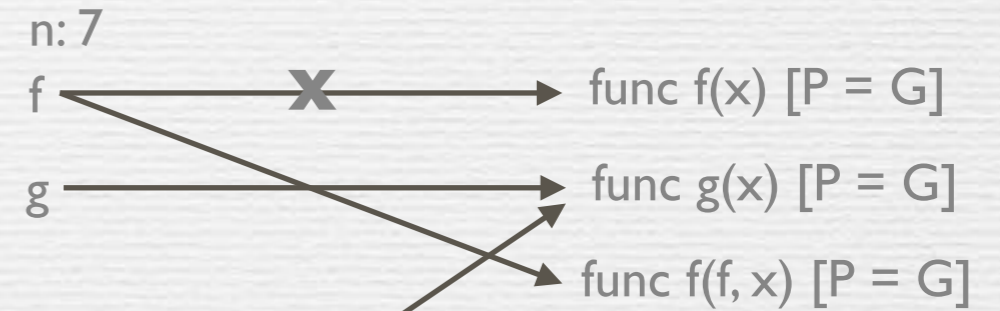
1.5 #2

```
→ n = 7
→ def f(x):
    n = 8
    return x + 1
→ def g(x):
    → n = 9
    → def h():
        ↓ return x + 1
    → return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]

f
x: 7

f2: g [P = G]

x: 14
n: 9
h: points to func h() [P = f2]

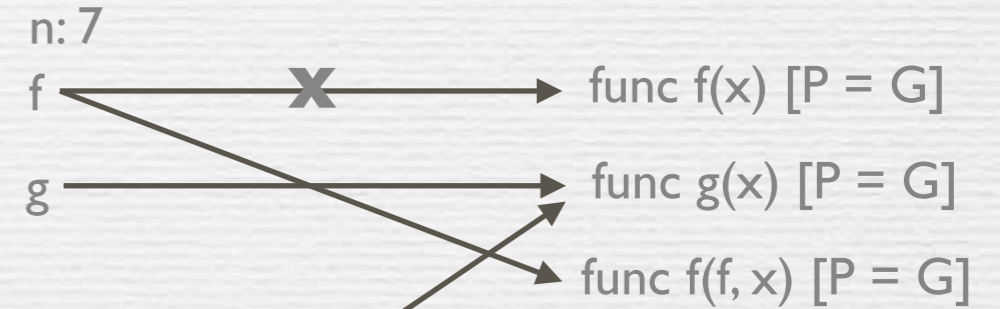
1.5 #2

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→ n = 7
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    → def h():
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    → return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]
f
x: 7

f2: g [P = G]
x: 14
n: 9
h → func h() [P = f2]
RV →

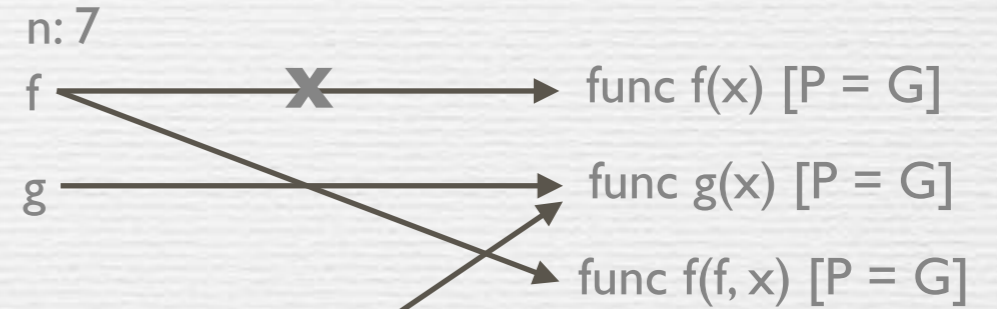
1.5 #2

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→ def f(f, x):
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```

```
→ f = f(g, n) function call
```

```
g = (lambda y: y())(f)
```

Global Frame:



f1: f [P = G]

f
x: 7
RV

f2: g [P = G]

x: 14
n: 9
h
RV

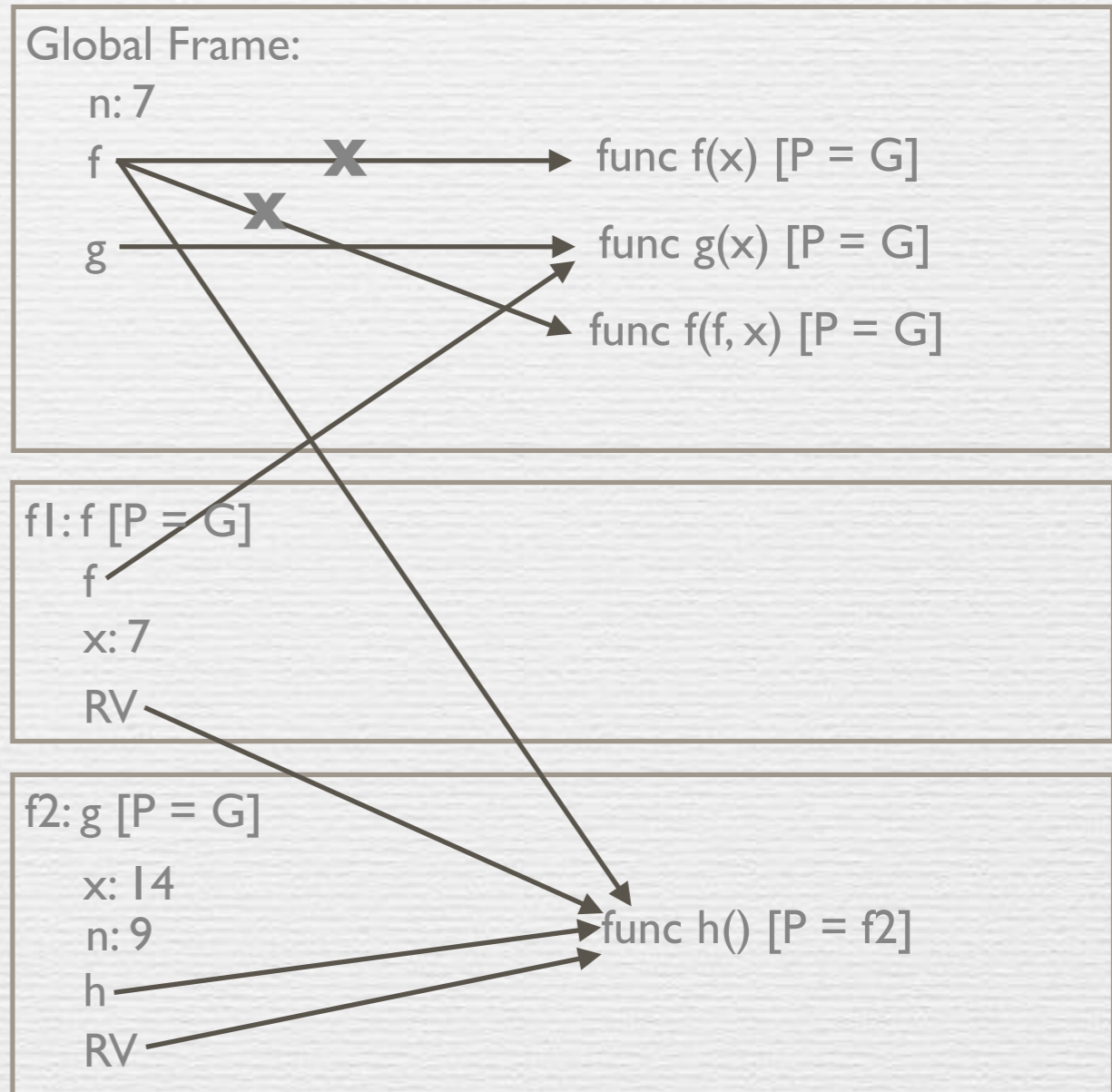
func h() [P = f2]

1.5 #2

```
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    return x + 1
→ def g(x):
    → n = 9
    → def h():
        ↓ return x + 1
    → return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call assignment
```

```
g = (lambda y: y())(f)
```

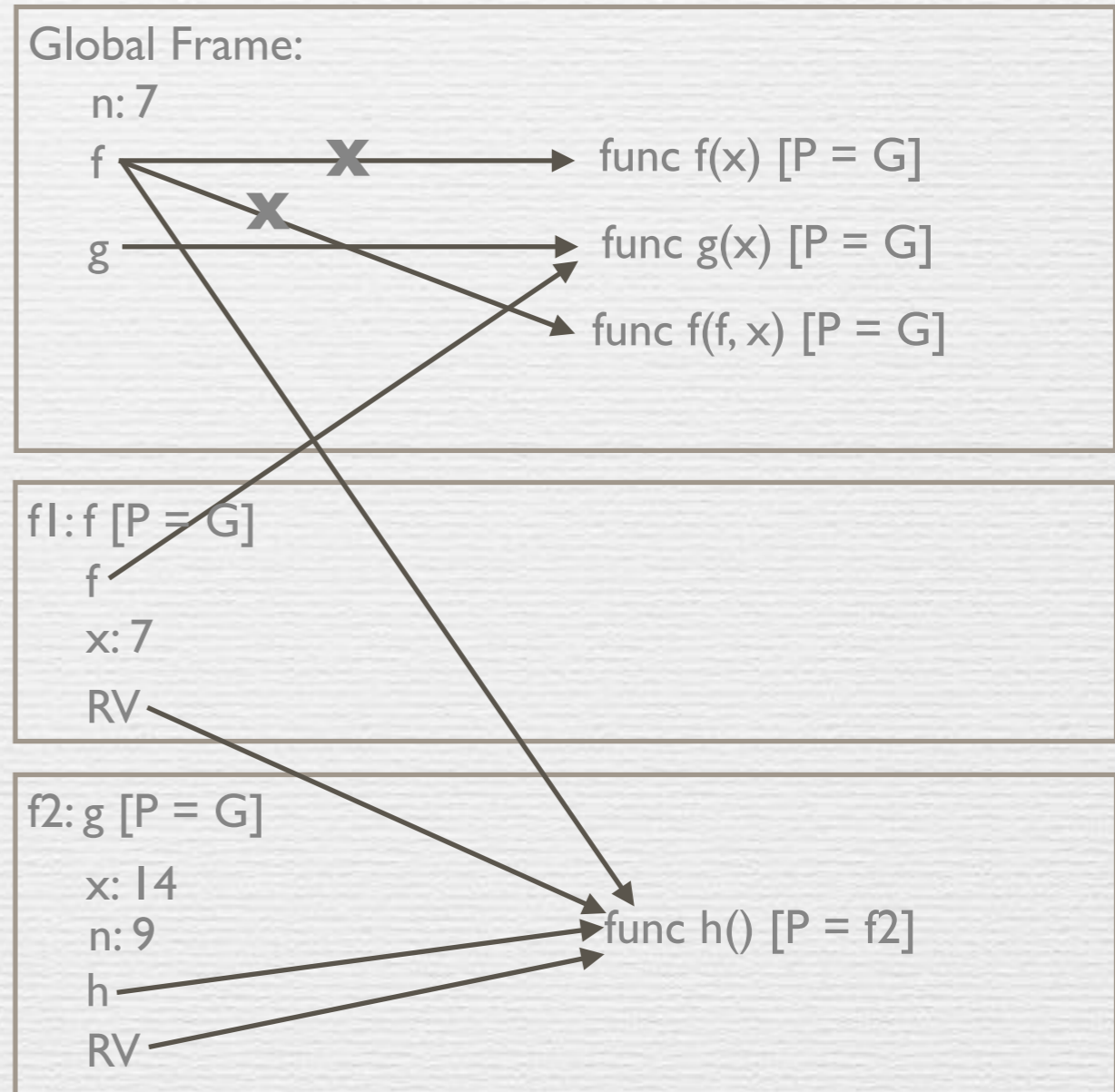


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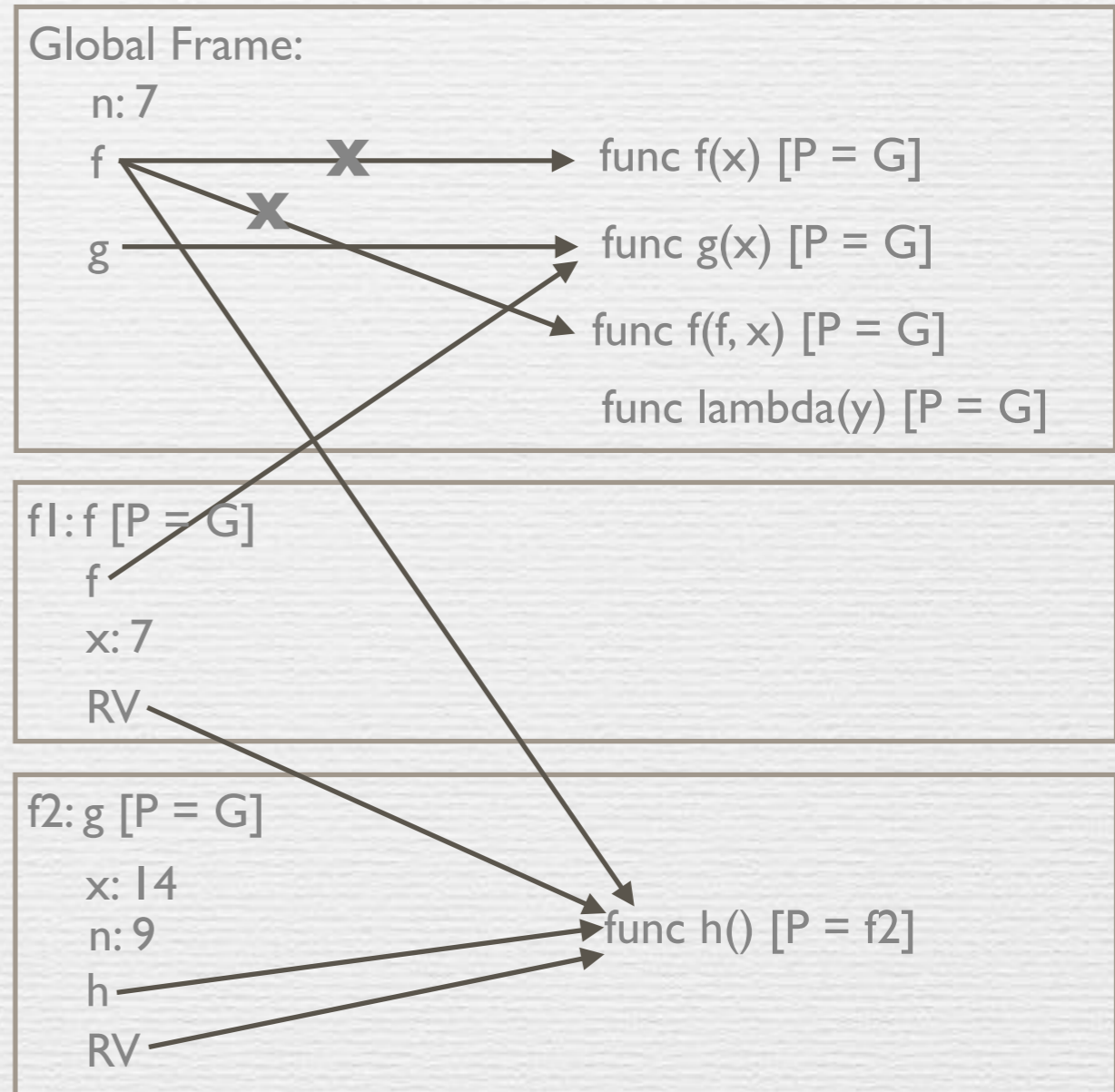


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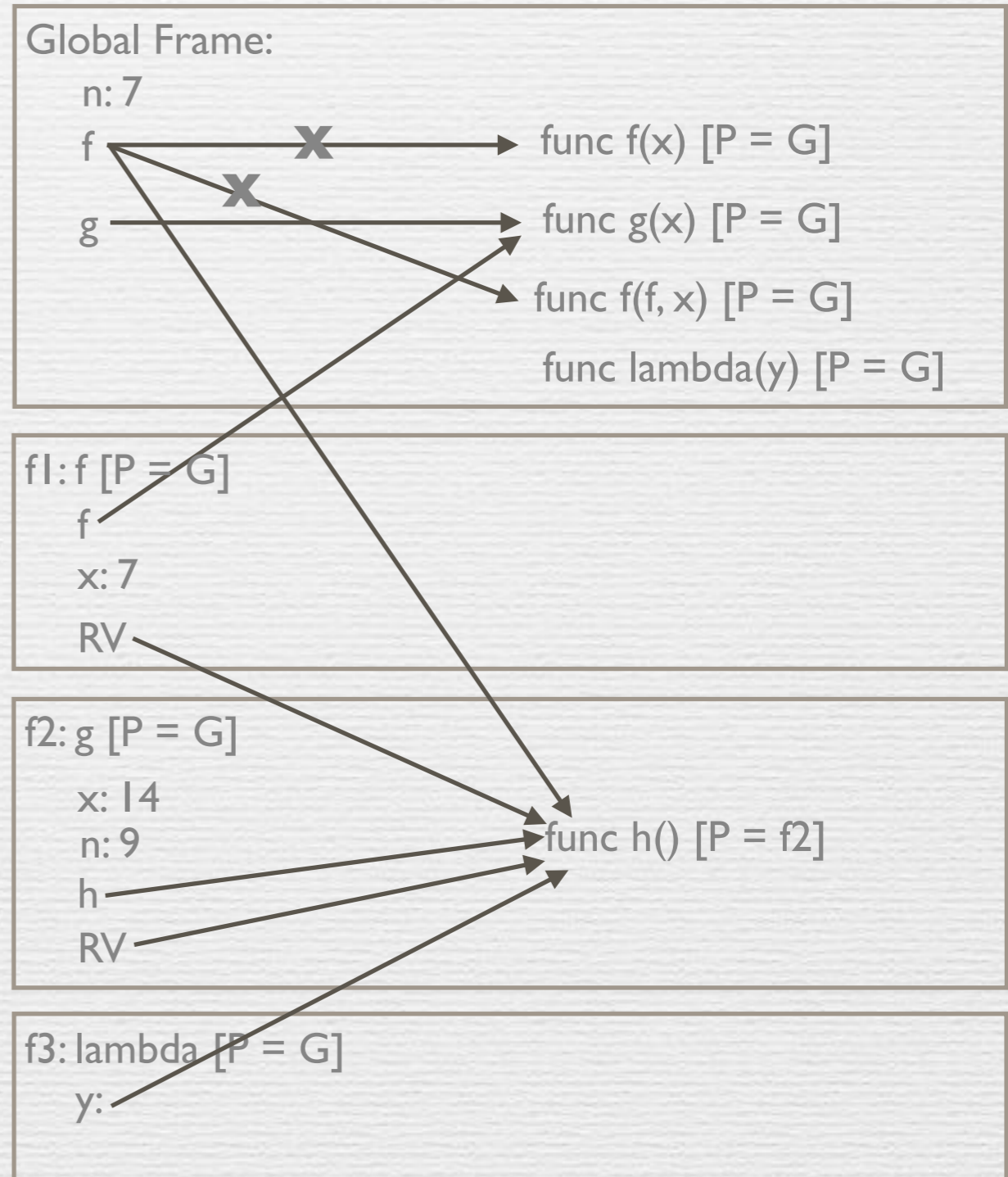


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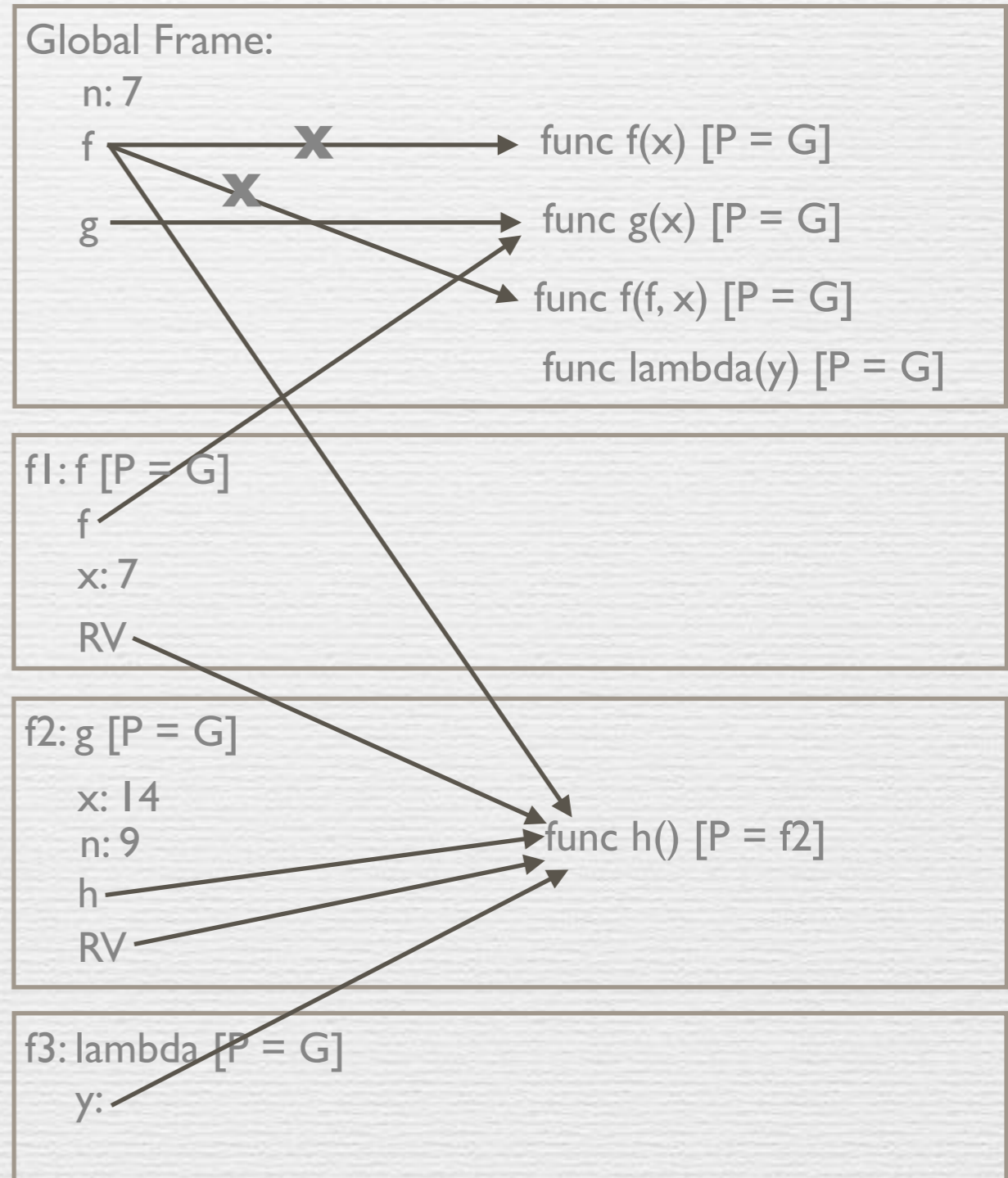


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→ f = f(g, n) function call assignment
```

```
→ g = (lambda y: y())(f) function call
function call
```

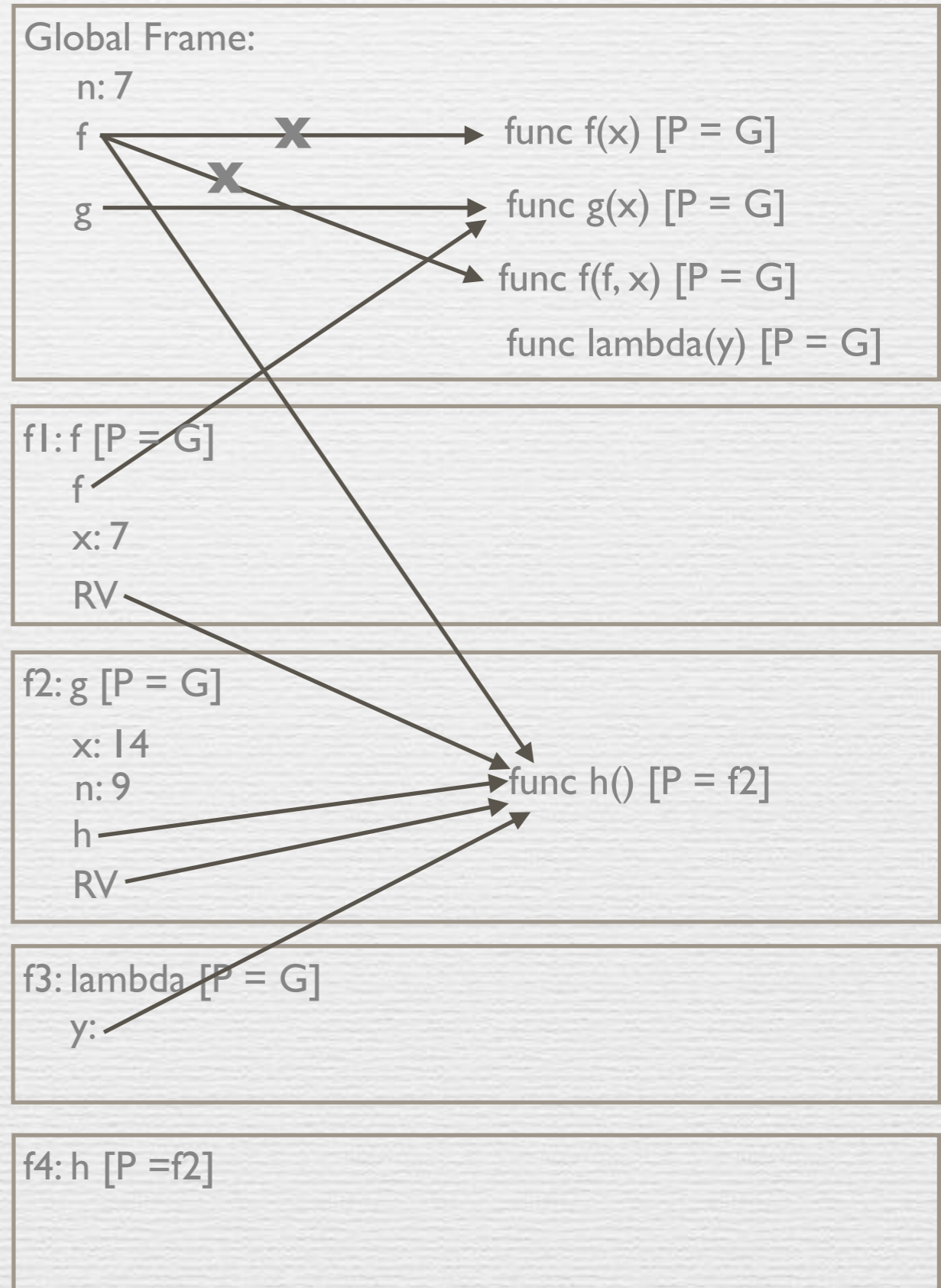


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    → n = 9
    → def h():
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→ def f(f, x):
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```
→ f = f(g, n) function call assignment
```

```
→ g = (lambda y: y())(f) function call
function call
```



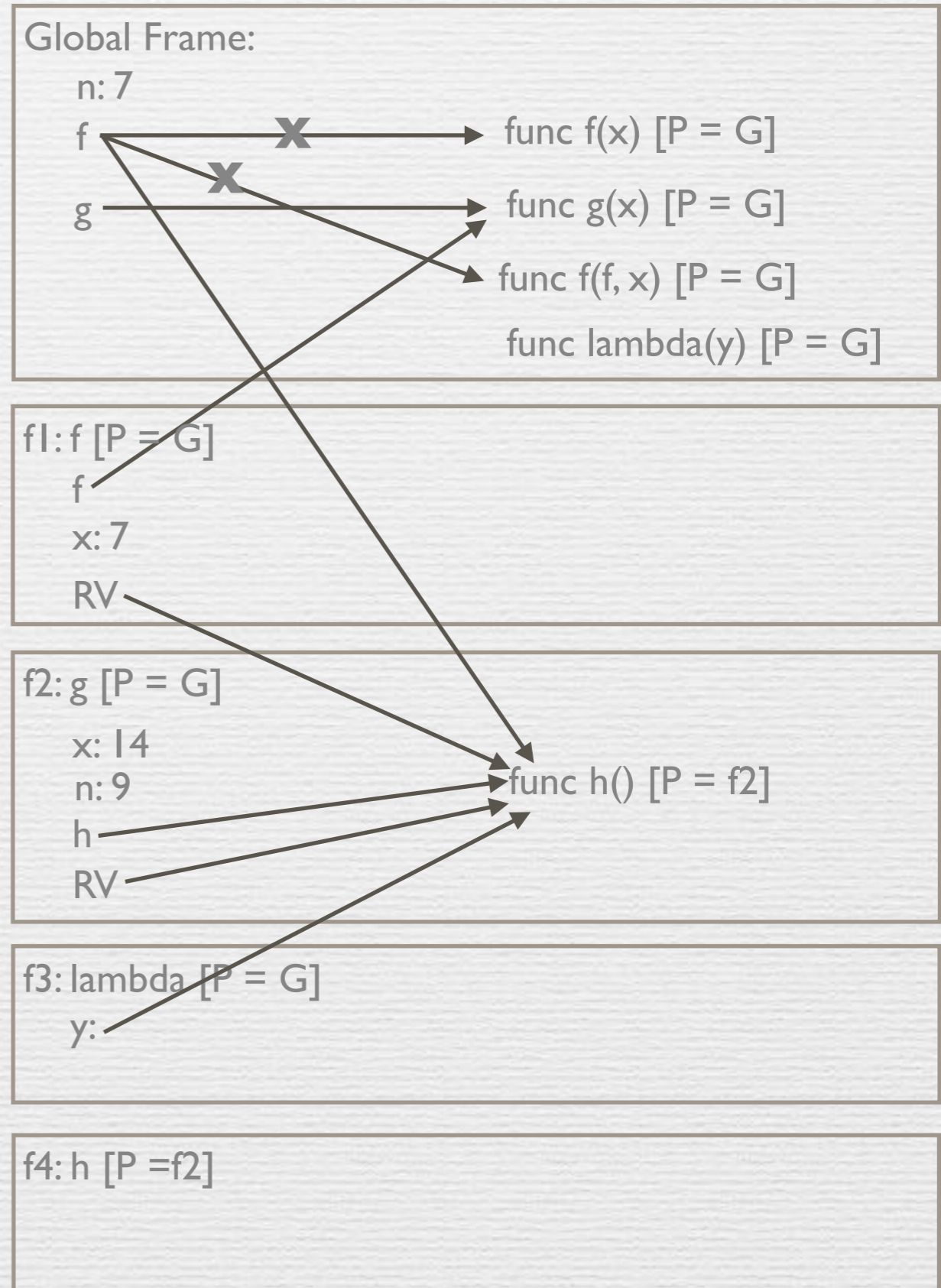
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→ def f(f, x):
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```

```
→ f = f(g, n) function call assignment
```

```
→ g = (lambda y: y())(f) function call
```

function call

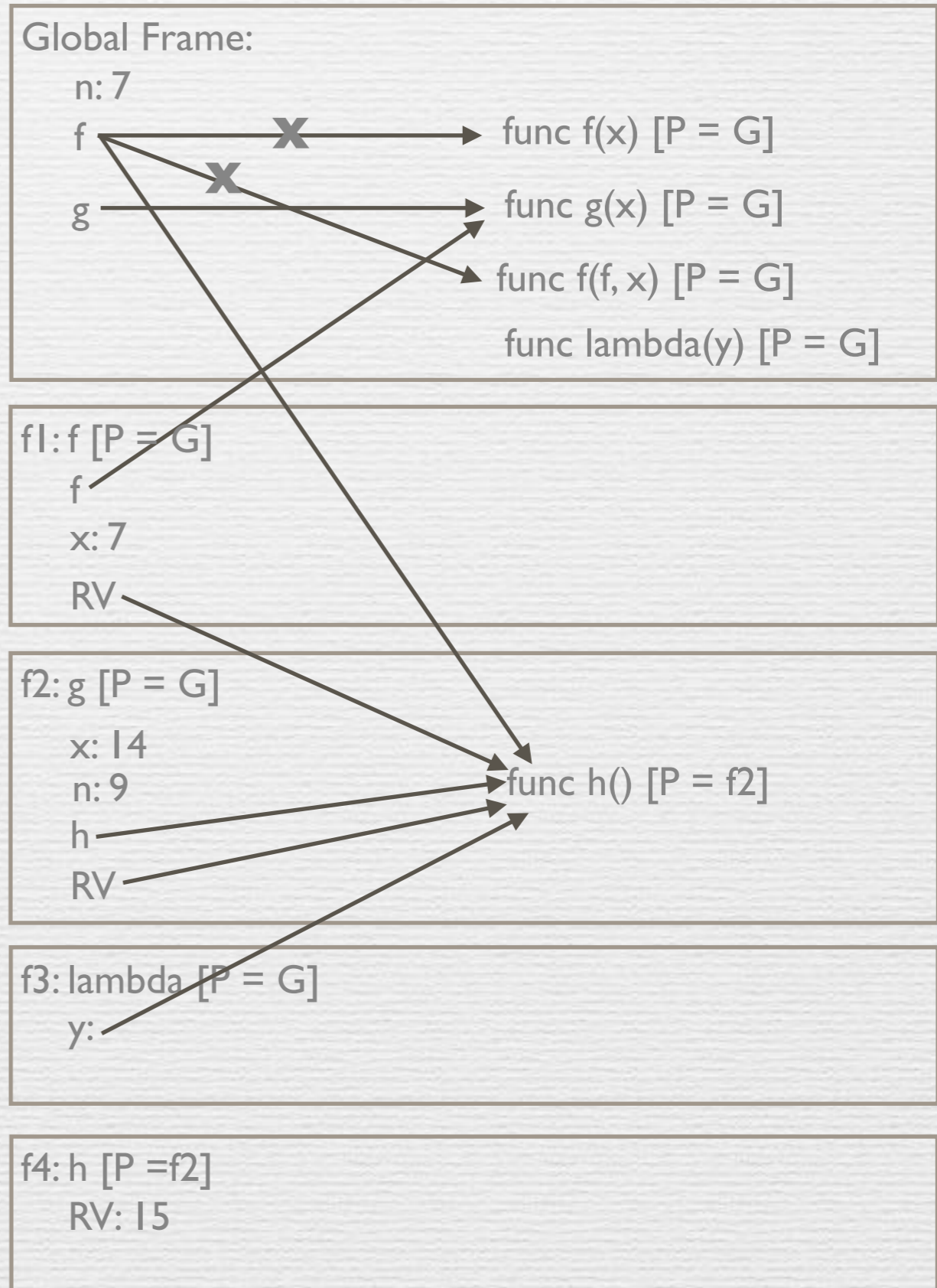


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→ def g(x):
    → n = 9
    → def h():
        → return x + 1
    → return h
→ def f(f, x):
    → return f(x + n) function call
```

```
→ f = f(g, n) function call assignment
```

```
→ g = (lambda y: y())(f) function call
function call
```

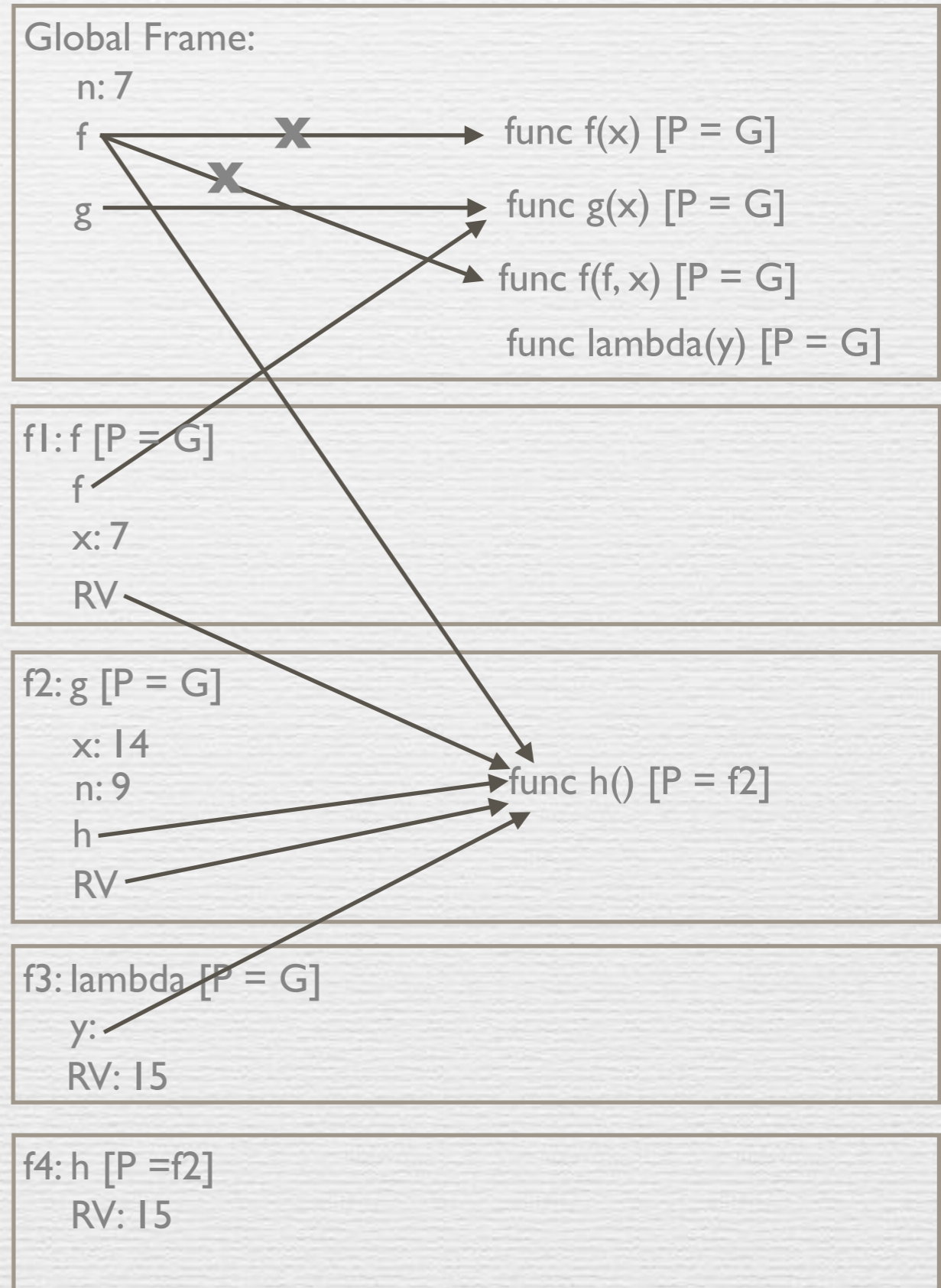


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→ def g(x):
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    → def h():
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→ def f(f, x):
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```
→ f = f(g, n) function call assignment
```

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function call
```



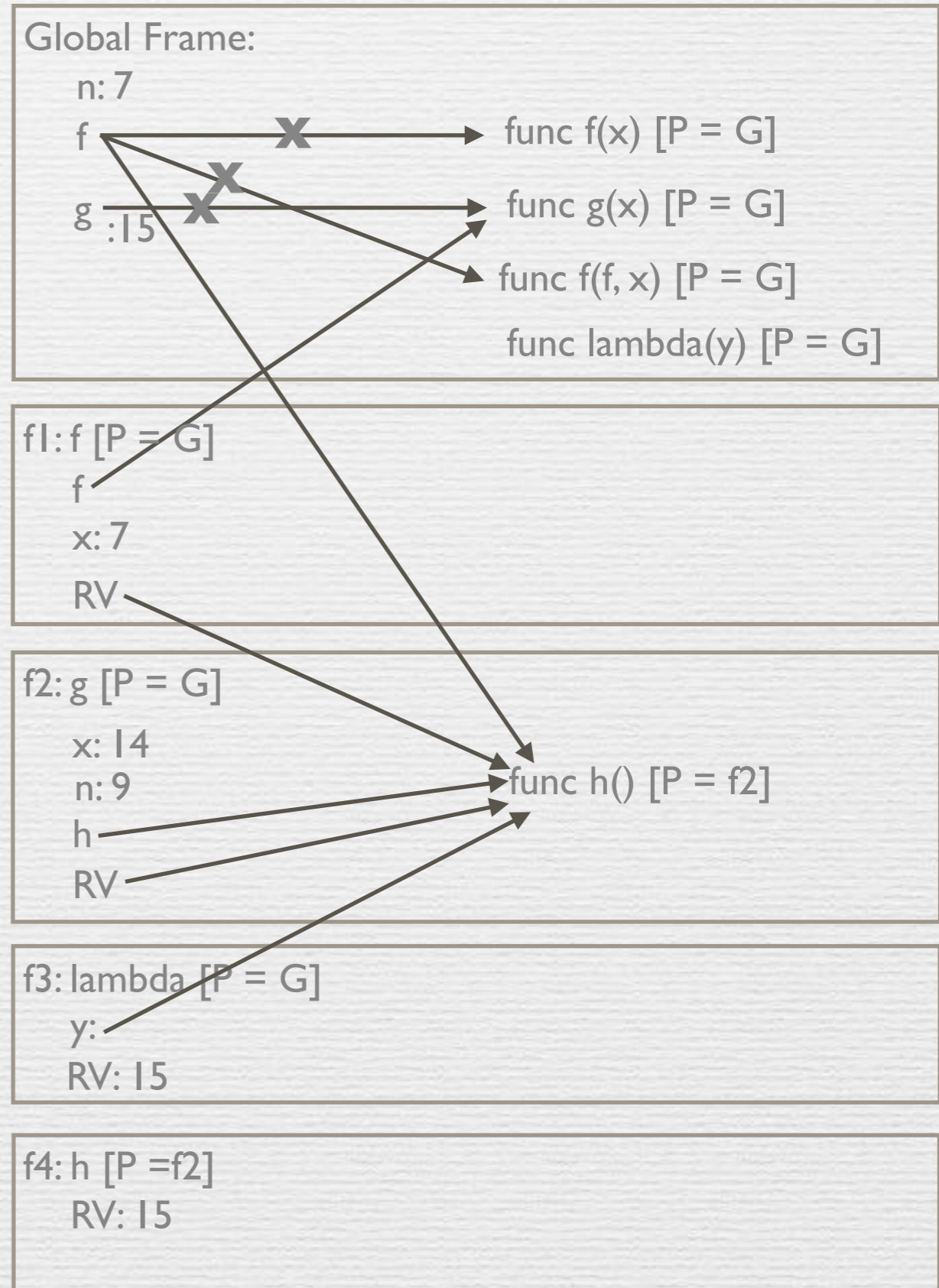
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```

```
→ f = f(g, n) function call assignment
```

```
→ g = (lambda y: y())(f) function call assignment
```

function call



Challenge Problem

1.5 #3

```
y = "y"
```

```
h = y
```

```
def y(y):
```

```
    h = "h"
```

```
    if y == h:
```

```
        return y + "i"
```

```
y = lambda y: y(h)
```

```
return lambda h: y(h)
```

```
y = y(y)(y)
```

1.5 #3

Global Frame:

y: "y"

→ y = "y"

h = y

```
def y(y):
```

```
    h = "h"
```

```
    if y == h:
```

```
        return y + "i"
```

```
    y = lambda y: y(h)
```

```
    return lambda h: y(h)
```

```
y = y(y)(y)
```

1.5 #3

→ `y = "y"`

→ `h = y`

```
def y(y):
```

```
    h = "h"
```

```
    if y == h:
```

```
        return y + "i"
```

```
    y = lambda y: y(h)
```

```
    return lambda h: y(h)
```

```
y = y(y)(y)
```

Global Frame:

y: "y"

h: "y"

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

`h = "h"`

`if y == h:`

`return y + "i"`

`y = lambda y: y(h)`

`return lambda h: y(h)`

`y = y(y)(y)`

Global Frame:

y: ~~'x'~~

h: "y"

func y(y) [P = G]



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

`h = "h"`

`if y == h:`

`return y + "i"`

`y = lambda y: y(h)`

`return lambda h: y(h)`

`y = y(y)(y)`

Global Frame:

y: ~~'x'~~

h: "y"

func y(y) [P = G]



1.5 #3

Global Frame:

y: ~~'x'~~
h: 'y' → func y(y) [P = G]

→ y = "y"

→ h = y

→ def y(y):

h = "h"

if y == h:

return y + "i"

y = lambda y: y(h)

return lambda h: y(h)

→ y = y(y)(y)

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

`h = "h"`

`if y == h:`

`return y + "i"`

`y = lambda y: y(h)`

`return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

`y: 'y'`

`h: "y"`

`func y(y) [P = G]`

`fl: y [P = G]`

`y:`

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

`if y == h:`

`return y + "i"`

`y = lambda y: y(h)`

`return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

y: ~~'x'~~

h: "y"

func y(y) [P = G]

fl: y [P = G]

y:

h: "h"



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`



`return y + "i"`

`y = lambda y: y(h)`

`return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

fl: y [P = G]

y:

h: "h"



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

`return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

y: ~~'y'~~

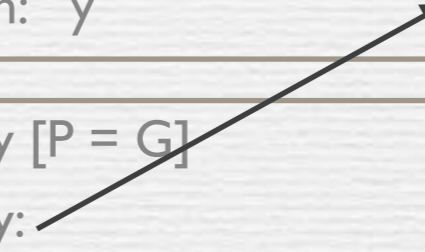
h: "y"

func y(y) [P = G]

fl: y [P = G]

y:

h: "h"



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

`return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

y: ~~'y'~~

h: "y"

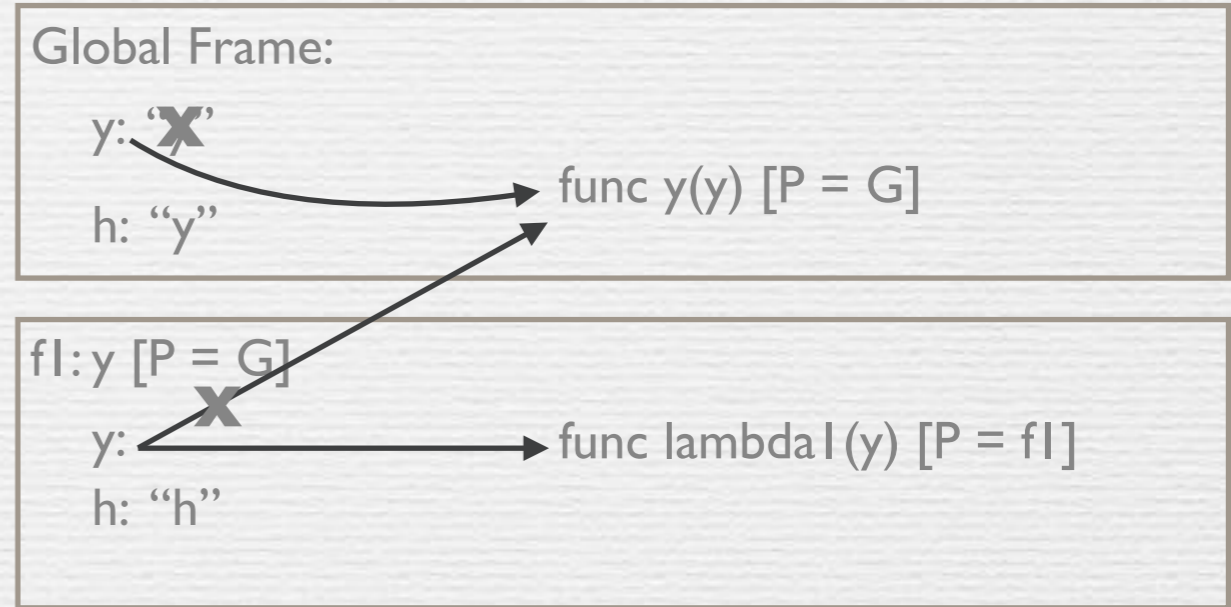
func y(y) [P = G]

fl: y [P = G]

y: ~~'y'~~

h: "h"

func lambda l(y) [P = fl]



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

→ `y = y(y)(y)`

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

fl: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = fl]

func lambda2(h) [P = fl]



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

→ `y = y(y)(y)`

We just completed the first `y(y)` function call: now we know what the **operator** is for the second function call

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

func lambda2(h) [P = f1]

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

→ `y = y(y)(y)`

We just completed the first `y(y)` function call: now we know what the **operator** is for the second function call

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

func y

"h"

`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

*y is not defined here, we must look for it in the **parent** frames h was passed in; in this case h is the function y (check f2)*

→ `y = y(y)(y)`

*We just completed the first y(y) function call: now we know what the **operator** is for the second function call*

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`



`return y + "i"`

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

*y is not defined here, we must look for it in the **parent** frames h was passed in; in this case h is the function y (check f2)*

→ `y = y(y)(y)`

*We just completed the first y(y) function call: now we know what the **operator** is for the second function call*

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

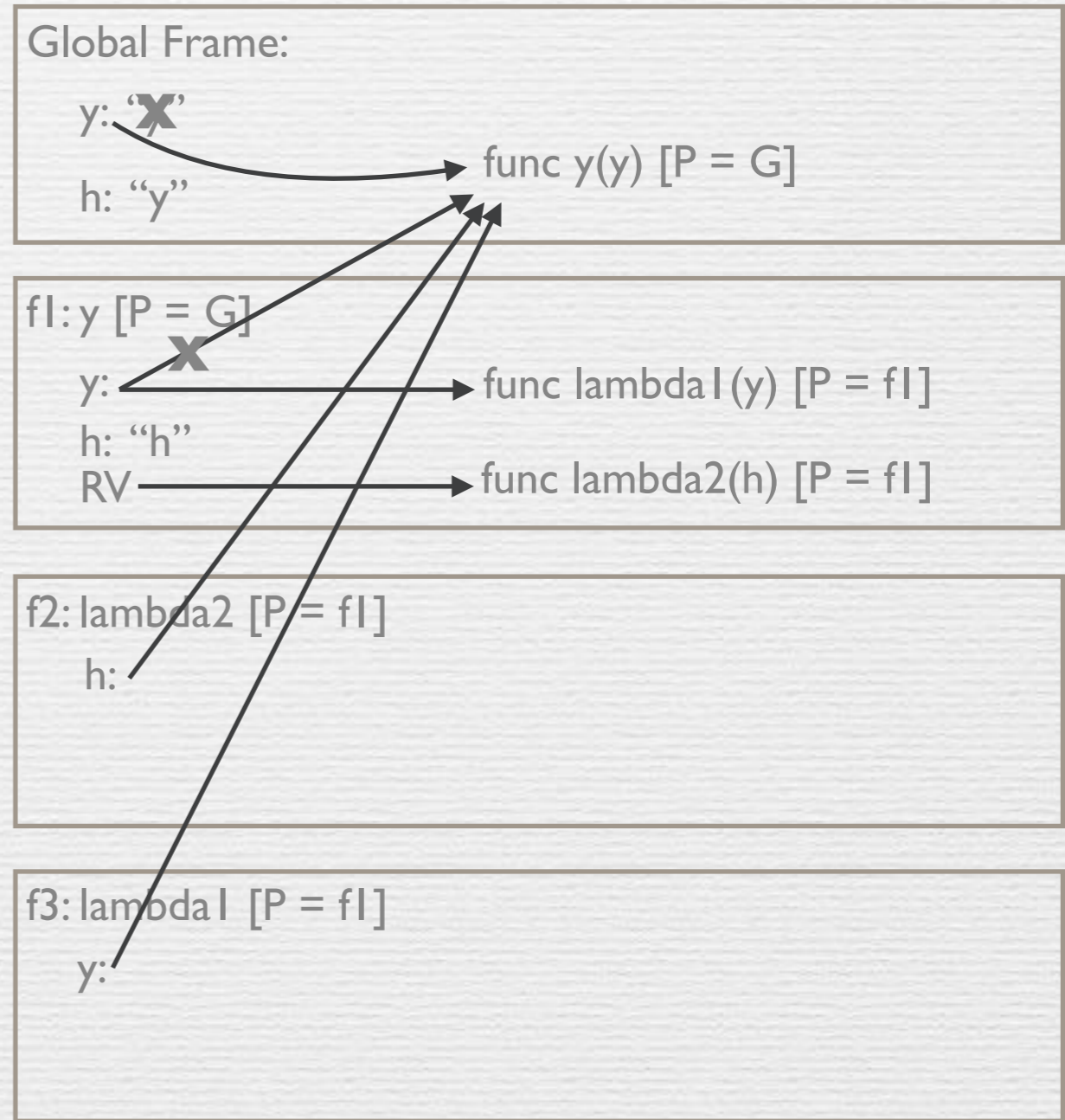
func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

f3: lambda1 [P = f1]

y:



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`

`return y + "i"`

→ `y = lambda y: y(h)`

→ `return lambda h: y(h)`

→ `y = y(y)(y)`

func y

"h"

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

assignment

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

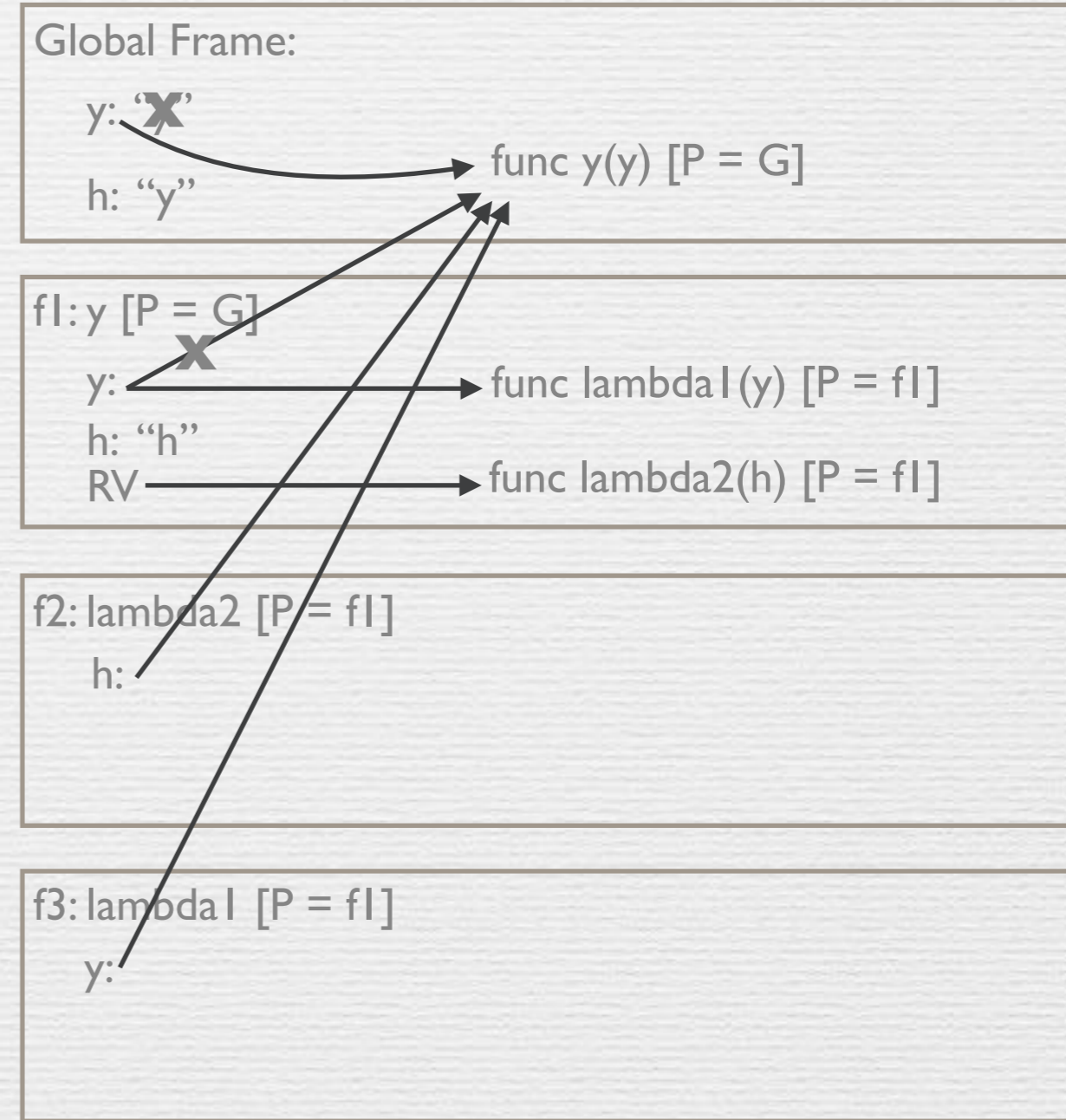
func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

f3: lambda1 [P = f1]

y:



1.5 #3

→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`



`return y + "i"`

→ `y = lambda y: y(h)`

assignment

→ `return lambda h: y(h)`

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

→ `y = y(y)(y)`

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~'y'~~
h: "y" → func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~ → func lambda1(y) [P = f1]
h: "h" → func lambda2(h) [P = f1]
RV → func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

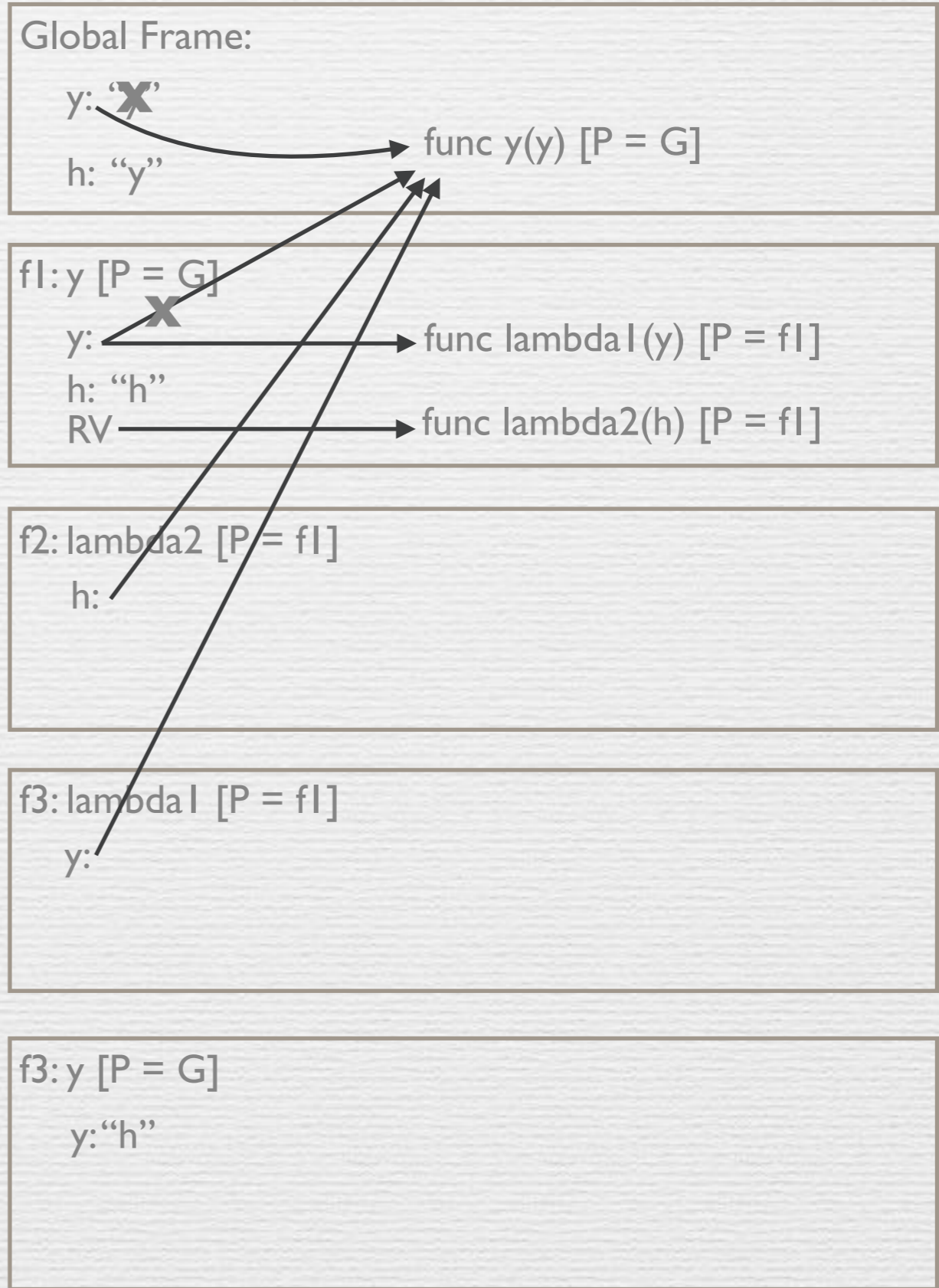
h:

f3: lambda1 [P = f1]

y:

f3: y [P = G]

y: "h"



1.5 #3

→ y = "y"

→ h = y

→ def y(y):

→ h = "h"

→ if y == h:

return y + "i"

→ y = lambda y: y(h) **assignment**

→ return lambda h: y(h)

→ y = y(y)(y)



The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~'y'~~

h: "y"

func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~

h: "h"

RV

func lambda1(y) [P = f1]

func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

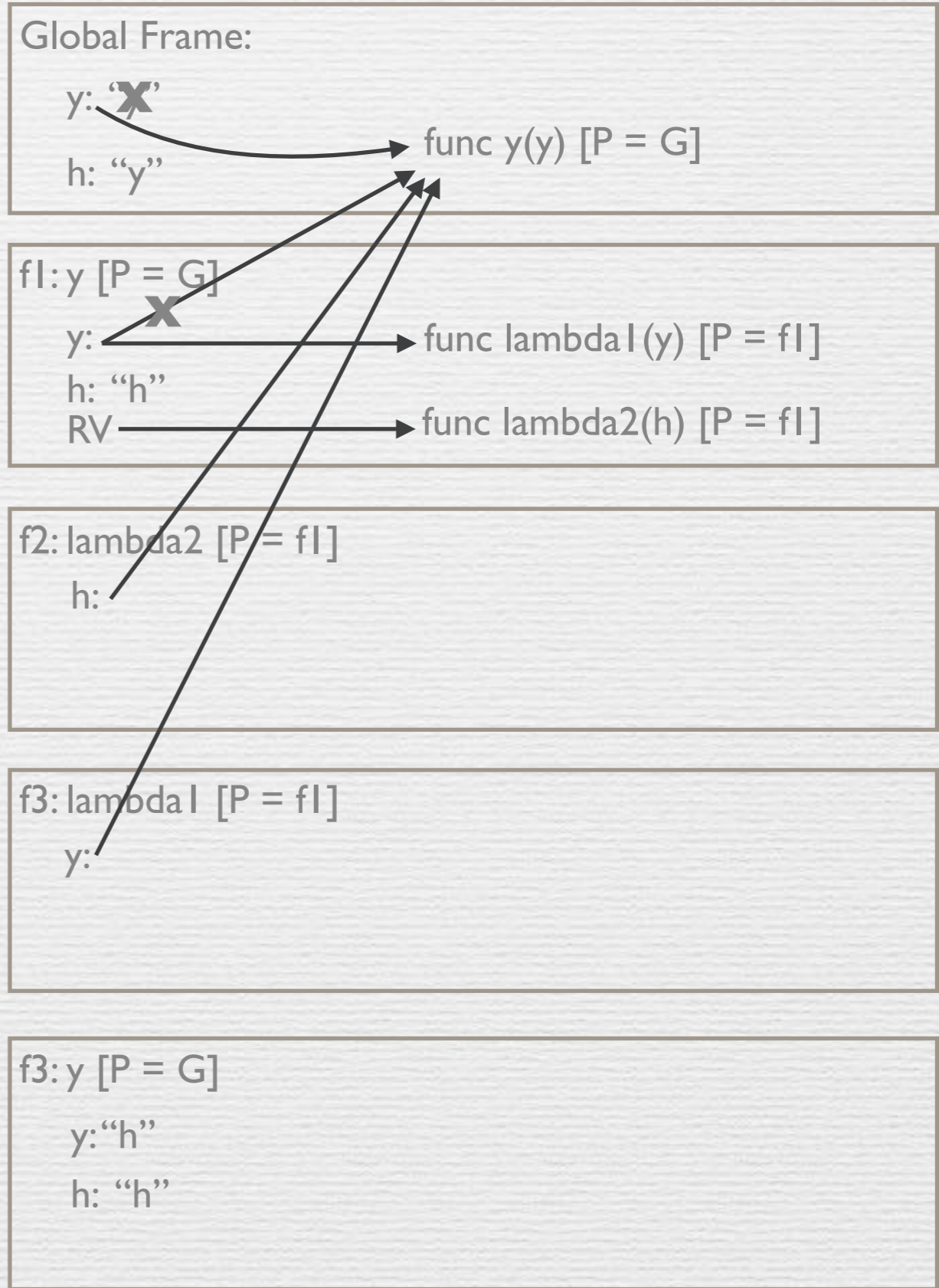
f3: lambda1 [P = f1]

y:

f3: y [P = G]

y: "h"

h: "h"



1.5 #3

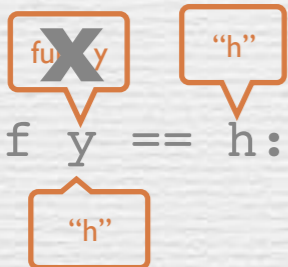
→ y = "y"

→ h = y

→ def y(y):

→ h = "h"

→ if y == h:



return y + "i"

→ y = lambda y: y(h) **assignment**

→ return lambda h: y(h)

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

→ y = y(y)(y)

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~'y'~~
h: "y" → func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~ → func lambda1(y) [P = f1]
h: "h" → func lambda2(h) [P = f1]
RV → func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

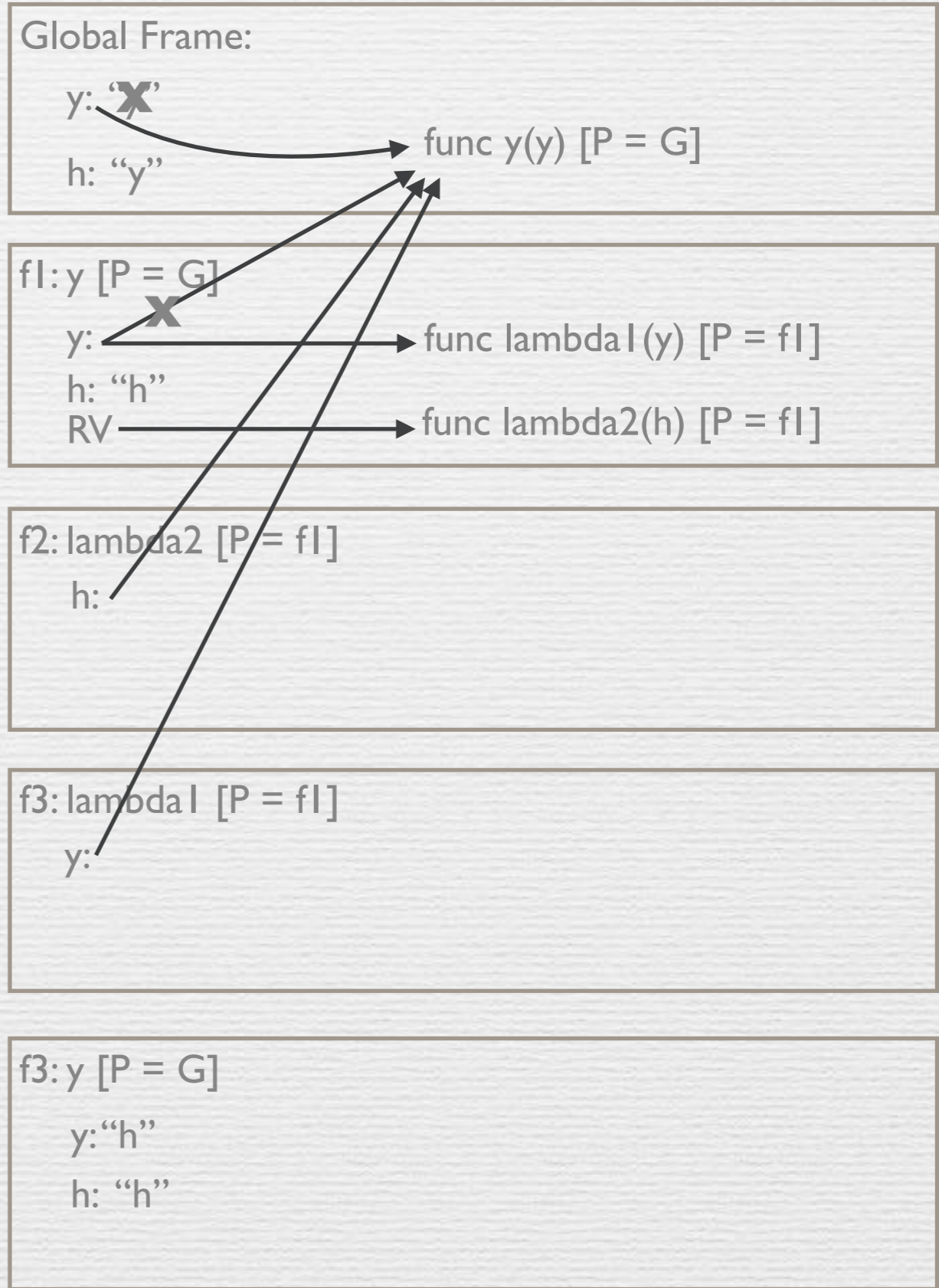
h:

f3: lambda1 [P = f1]

y:

f3: y [P = G]

y: "h"
h: "h"



1.5 #3

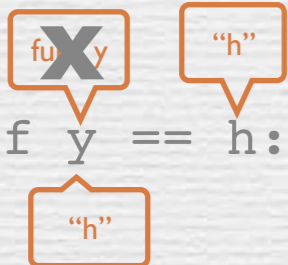
→ y = "y"

→ h = y

→ def y(y):

→ h = "h"

→ if y == h:



→ return y + "i"

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

→ y = lambda y: y(h) **assignment**

→ return lambda h: y(h)

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

→ y = y(y)(y)

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~'y'~~
h: "y" → func y(y) [P = G]

f1: y [P = G]

y: ~~'y'~~ → func lambda1(y) [P = f1]
h: "h" → func lambda2(h) [P = f1]
RV → func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

h:

f3: lambda1 [P = f1]

y:

f3: y [P = G]

y: "h"
h: "h"
RV: "hi"

1.5 #3

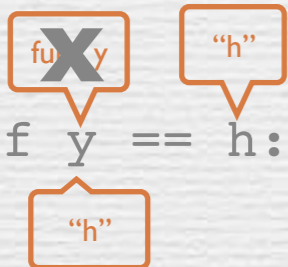
→ y = "y"

→ h = y

→ def y(y):

→ h = "h"

→ if y == h:



→ return y + "i"

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

→ y = lambda y: y(h) **assignment**

→ return lambda h: y(h)

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

→ y = y(y)(y)

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~X~~
h: "y" → func y(y) [P = G]

f1: y [P = G]

y: ~~X~~ → func lambda1(y) [P = f1]
h: "h" → func lambda2(h) [P = f1]
RV →

f2: lambda2 [P = f1]

h:

f3: lambda1 [P = f1]

y:
RV: "hi"

f3: y [P = G]

y: "h"
h: "h"
RV: "hi"

1.5 #3

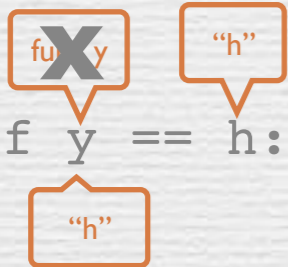
→ `y = "y"`

→ `h = y`

→ `def y(y):`

→ `h = "h"`

→ `if y == h:`



→ `return y + "i"`

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

→ `y = lambda y: y(h)` **assignment**

→ `return lambda h: y(h)`

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

→ `y = y(y)(y)`

We just completed the first y(y) function call: now we know what the operator is for the second function call

Global Frame:

y: ~~X~~
h: "y" → func y(y) [P = G]

f1: y [P = G]

y: ~~X~~ → func lambda1(y) [P = f1]
h: "h" → func lambda2(h) [P = f1]
RV → func lambda2(h) [P = f1]

f2: lambda2 [P = f1]

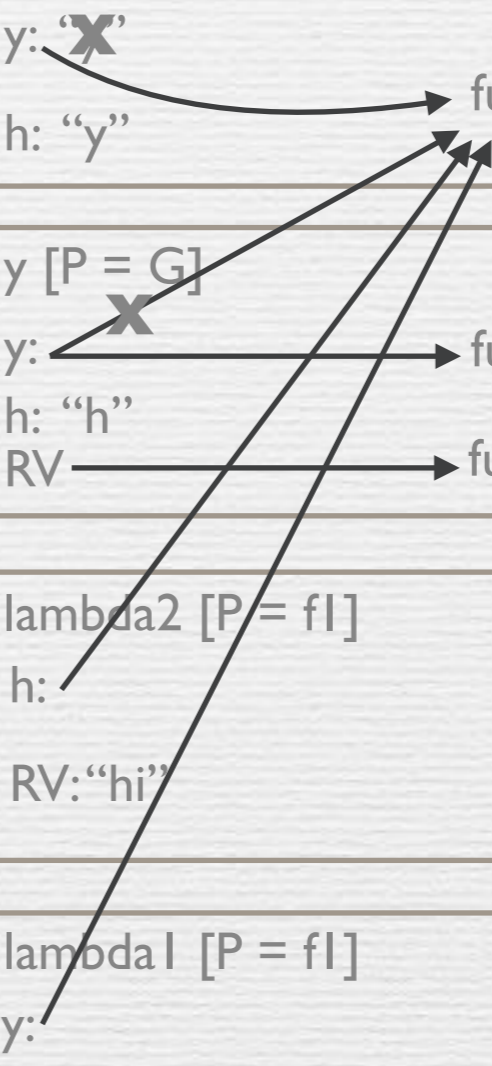
h:
RV: "hi"

f3: lambda1 [P = f1]

y:
RV: "hi"

f3: y [P = G]

y: "h"
h: "h"
RV: "hi"



1.5 #3

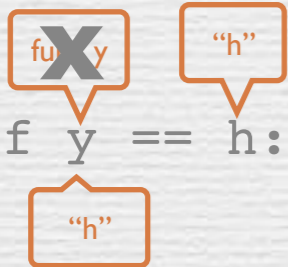
→ y = "y"

→ h = y

→ def y(y):

→ h = "h"

→ if y == h:



→ return y + "i"

The return value of lambda1 is the result of calling y (which is what we passed in) on h. Since h is not defined in this frame, we must look at the parent frames

→ y = lambda y: y(h) **assignment**

→ return lambda h: y(h)

y is not defined here, we must look for it in the parent frames h was passed in; in this case h is the function y (check f2)

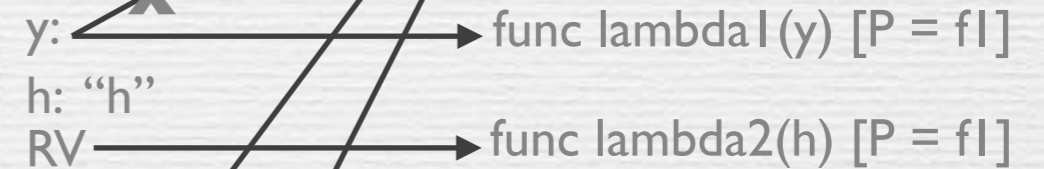
→ y = y(y)(y)

We just completed the first y(y) function call: now we know what the operator is for the second function call

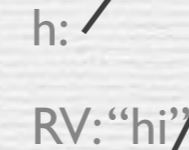
Global Frame:



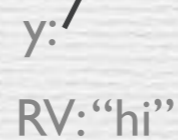
f1: y [P = G]



f2: lambda2 [P = f1]



f3: lambda1 [P = f1]



f3: y [P = G]

